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says
Mr Earl is already well known as
the author of the admirable
"Practical Lessons in physical
measurement".

PRACTICAL LESSONS IN PHYSICAL MEASUREMENT

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IN

PHYSICAL MEASUREMENT

BY

ALFRED EARL, M.A.

LATE SCHOLAR OF CHRIST'S COLLEGE, CAMBRIDGE; OF THE INNER TEMPLE
BARRISTER-AT-LAW; SENIOR SCIENCE MASTER AT TONBRIDGE SCHOOL

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PREFACE.

The Laboratory Course contained in the following pages is intended as an introduction to the serious study of Science, and is the outcome of my experience as to the methods suitable to beginners. It is meant for use in schools provided with means of practical work, and has been written in the belief that it may be of help to others than myself interested in the teaching of Science, others, who are assured with me, that the Scientific Sides of Public Schools should not be devoted simply to instilling into boys a certain amount of technical knowledge, but should rather train them to observe accurately, to reason rightly, and to front nature with an open and inquiring mind. With this end in view care has been taken to make the course logically progressive; and more attention has been given to the methods employed in observation than to the actual value of the facts revealed by those methods. The book deals only with measurements of length, mass, and time, and with such of these measurements as are of the most elementary kind; but small as is the region of knowledge explored, and simple as the plan of the book may seem, the methods employed and taught therein are those which will be called into play in any research, however abstruse, which the student may hereafter undertake.

Nothing is so disappointing and so baffling to a teacher as the jumble of disconnected knowledge which is often found to exist, without the reasoning power and logical grasp which is needed for its mastery, in the mind of the tyro who has gone ahead too rapidly. Passing, too, from school to a wider field, it is evident that the quickly growing mass of scientific facts is becoming very heavy for the framework of law which supports it. This being so, it must be admitted more generally than it is customary to do, that the retention of facts should be subordinate in scientific education to a sound comprehension of them, that a mind which has been trained to observe and compare accurately is most likely to acquit itself well in the world, and, lastly, that a training in physical measurement is the most solid basis of scientific knowledge. When this is recognized, Science may hope to take her right place in education. At the same time one must admit that in

secondary education at any rate, which of the recognized subjects is taught, is not of so much importance as the manner in which it is taught. The right way of learning is chiefly to be cultivated. In learning science the matter is obviously less important than the method. A logical and inquiring habit of mind is more valuable than the memory of facts and laws. It is a better equipment for future research and knowledge. Moreover it ought to be found that some training in correctness of expression and accuracy of language is gained from such a course as follows. This is no insignificant part of scientific education, although it is seldom recognized.

These ends I have kept in view. Whether the means are adapted to the ends must be judged from the general tendency of the scheme, bearing in mind that it is an introductory course. In connection with this subject one may be permitted to notice what is often prominent as a defect, especially in the "modern sides" of schools, a lack of continuity in work. There is little doubt that much of the average "capacity of learning" is never utilized owing to frequent change both of subject and master, which seems a tradition of many modern sides. Change of subject is necessary and useful, but an "overlapping" of subjects tends to confusion of thought. Perhaps this book may serve in some degree to bridge over with safety the distance between the laboratory and other class-rooms by acting as a "Practical Arithmetic," and, to some extent, as a "Practical Grammar."

I should be ungrateful if I failed to acknowledge the assistance which has been very kindly given by my old pupils, Mr. C. C. Roberts, B.A., Scholar of Christ's College, Cambridge; Mr. V. H. Jackson, Scholar of Balliol College, Oxford, and Mr. E. R. Clarke, Scholar of St. John's College, Cambridge.

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CHAPTER I.

INTRODUCTORY.

WHAT NATURAL SCIENCE MEANS, AND WHY WE LEARN IT.

1. EACH one of us is aware of a separation of himself from the rest of the world. There seems to be a division of life into two parts—very different and very unequal parts: self on the one hand, and all that which is not self on the other. Within ourselves are feelings and thoughts; outside ourselves the varied and wonderful universe. And each individual stands a centre for himself of all things, knowing that around him are objects of endless variety, and that the further he explores in any direction more and more of the world will open out to him. He can think of himself as a centre from which start forth in all directions paths leading to knowledge of the world. It is by way of these paths that he learns about **Nature,** for so we call the Universe; and the knowledge acquired is called **Natural Knowledge or Natural Science.**

2. What are these paths which put us into communication with nature, and along which knowledge travels to us, and by which we advance to knowledge? They are the five senses—**Sight, Touch, Hearing, Smell,** and **Taste.** To render the picture complete, we must fancy this self of ours making journeys of discovery along the paths of the senses. Now it will travel easily and reach remote parts, as on the path of sight, one used more frequently than any other. At another time the path of hearing, though it does not lead far afield, enables us to penetrate deeply into certain parts of nature; and though we use the other three paths less often, they have each their own value as roads to our knowledge of the outside world. But the sense of touch deserves some special mention, as we may learn by it much more about nature than by smell or taste. Difficult as it seems to believe, it is still true, that touch can sometimes inform us more accurately about the world within our reach than sight can do, and that without touch our science would be often found wanting.

3. You must not suppose that what has been said here contains more than the smallest fraction of all that is to be learnt of our relation to the world in which we live; but you must be content at present to be told and to believe that these matters have been pondered over and discussed by the learned men of all ages, without any end to the discussion having been reached. Such high things as intelligence, instinct, reason, thought, their place and work in the world, we must leave to be considered by those who are able to deal with them, the men to whom has been given the name philosopher, that is, lover of wisdom. We must be content *for the present* to regard nature as capable of being approached and

understood solely through the **five senses.** Afterwards, we may learn to think of nature under other aspects, and approach it by other paths.

4. The meaning of natural science is knowledge of nature, and the aim of the student of science is to learn all he can about nature. You may think that as we learn natural facts through our senses, we must have been learning natural science, whether consciously or unconsciously, from the moment we began to use our senses, that is, from our birth; and that is true; but it is not the whole truth. If it were, and our own senses were all we had to depend on for knowledge of nature, then the people who had the best sight, and touch, and hearing would know the most of nature; and the standard of knowledge among us would be much the same for every one. The answer to this is, that we do not rely solely on the evidence of our own senses, but are aided by vast stores of knowledge gained through the observation and intelligence of those who have been before us, and of those who are at work among us now. **These stores of knowledge are recorded in books,** which we can all obtain, and through which any discovery gained from nature becomes the common property of the whole human race. If you can understand this rightly—can realize how the world has been enriched by those who have left us these treasure-houses of knowledge, raised by their laborious days for our use and profit, you will always revere books. Without books and education we should be deprived of all knowledge of nature beyond what our own senses can teach us, and that is no large amount.

5. Here, then, we see the meaning of **education** or **training**—on the one side is the learner capable through his senses of observing and discovering something for himself; but this indeed is as nothing compared with that vast mass of knowledge which fronts him on the other side, the result of the ceaseless efforts of innumerable workers. It is the part of education to lead him too into their ranks, to teach him to use his own slender stock of knowledge, gained straight from nature, in such manner as shall place him in touch with the experience of ages; and so his knowledge grows apace. The knowledge is there, far more than any one can hope to make his own; the only question is, how much of it he can acquire; and the only object of education is to enable him to acquire as much as is possible, and fitting, and worthy to be acquired.

6. If it were enough in regard to science to be told facts and to remember them, then all that would be needed to deserve the name of a student would be a good memory; and the better the memory and the more things remembered, the better the training. But to imagine this would be a great mistake, preventing your science ever being worthy its name. It is little use to be told or to remember facts, unless you can understand them, that is, see what other facts they lead to, and how they form part of a general plan. And these first pages have been written to make you master of this fundamental truth—**that in study, method is first, and facts are second in importance; that the great need is to learn how to learn.** If the student misunderstand the one small tract of knowledge on which he is working, what

chance is there that he will comprehend rightly the rest of nature? But one small province, approached and made your own by right methods, will be a firm standing place from which advances may be safely made into further fields. When we receive a training in science we do not learn much—much is not even attempted; but what is learnt must be learnt in the right way.

7. You need not be told that the knowledge of nature has many sides not included in the term natural science or *science* (as it is often called for shortness), which are to be found under the name of philosophy, history, classics, mathematics—all that our forefathers used to call the "liberal sciences." Therefore you will not suppose that the student of science is learning all there is to be known of nature. But, in studying science in the right manner, he is adding to his power of learning and is strengthening all his mental faculties; so that, if he will, he may take to any other subject the more easily. He will achieve this by selecting some small department of knowledge, and observing the methods by which this knowledge has been accumulated; or he will take some corner, as it were, of nature and note all he can therein, using in his inquiry such methods as may be applied and extended to any development of his subject. The propriety of the methods employed will decide the failure or success of his efforts; and on the selection of the best methods of acquiring knowledge the minds of many of the wisest men of all times have been exercised.

8. Hence it will be our aim in the following course to learn accurately something at least of nature, but the more important intention will be to learn the right methods of observation, or, in other words, how to use our senses to the best advantage. It is perhaps as well to understand distinctly at the beginning that the inquiry into the *causes of events* belongs to a later stage of study, and perhaps we may never discover them. We have to begin by learning how to observe or perceive accurately; and this is not so easy as it seems. Afterwards we make experiments; learn from the records of other observers; and also use our reasoning powers, in order to extend our knowledge much more widely than we could do by simple observations. **But it is just because observation is the basis of science that it must be as accurate and certain as human power can make it.**

In order to observe we must compare. Observation begins with comparison or bringing together, and we cannot compare one thing with another without going through some kind of measurement, even although the measurement may be so rapid that we are not aware that it has taken place. **Hence it is that the real beginning of exact knowledge, or science, lies in measuring, and a faithful observer of nature is always occupied in measurement.**

9. ARGUMENT.—There is a world within ourselves and a world without.—The world or universe without is made known to us through the senses.—A knowledge of nature as perceived by the senses is the province of natural science.—Our knowledge is not limited to what we ourselves have perceived.—We are able to refer to the records of other observers.—The stores of accumulated knowledge are not to be approached by memory alone, but with the aid of our own stock of

experiences.—It is the aim of education to put us in touch with the observations and thoughts of the wisest men.—Hence we are made to learn the right methods of observing nature.—Thereby we can proceed by easy stages from the knowledge gained by ourselves to the wider knowledge built up by others; our methods, if correct, being capable of leading us to further truths.—It is important to remember that science regards nature from one side only.—Observation begins with comparison.—Comparison implies measurement.—The beginning of science lies in measurement.

CHAPTER II

MEASUREMENT OF LENGTH.

DESCRIPTION OF METHOD EMPLOYED.

10. The simplest kind of measurement is that of **length** or **distance**. At any rate, it is very easy to measure distances which do not require to be known with the utmost accuracy, or to measure in such a way that some knowledge of distance is obtained. For example, a yard of cloth or the length of a room can be very easily measured with as great an accuracy as may be required.

But even simple and easy operations of this kind are carried out **on the same plan** as those which require great skill and the use of delicate instruments.

It is necessary to have something with which to measure. All measurements are carried out by the use of a *standard*, as it is called.

The actual instrument may not be a standard quantity, but an object containing the standard many times for convenience of measurement, for example, a tape or chain measure.

The standard is laid alongside the length to be measured, moved along it, and by the *sense of sight*

we ascertain how many lengths equal to the standard are contained in it.

We may say that **a standard length** is a fixed and invariable length used in measuring other lengths. Later on, standards will be spoken of as *units*. The important point is to remember, that in order to measure any quantity we require **another quantity of the same kind.**

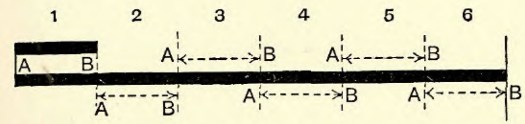


FIG. 1.—The above diagram shows the process of measuring an unknown quantity, namely, the length of the long thick line, by means of a standard quantity, the length of the short thick line. The measured length is seen to be six times the standard.

11. Anything which can be expressed as so many times a standard, or such a fraction of a standard, is a quantity.

There are many kinds of quantities which can be measured, but all of them have to be measured by the same *method* as length is measured, although the same means may not always be used; **and all of them have to be expressed as so many times the standard quantity, just as length is so described.**

EXERCISE.—Write out in your own words the meaning of the words *standard* and *quantity*, and describe how the measurement of a length is made.

EXERCISES ON MEASUREMENT OF LENGTH.

N.B.—*An account of each experiment must be carefully written out in a note-book. Wherever needed, drawings are to be made.*

1. Measure the size, or dimensions, in inches, of the paper on which you are writing, and write out the results, as follows :

Length of sheet of paper, - - -	$12\frac{1}{4}$ inches.
Breadth " " - - -	$8\frac{1}{2}$ "

Use for your standard a strip of paper, one inch in length, which you have divided into halves, quarters, and eighths.

2. Judge by eyesight the length and breadth of the top of the work-bench, and then compare the results of your judgment with those obtained by using a lath one foot long, which you have divided into inches.

Note that in the first case a rapid and rough measurement is taken by means of the eyesight, together with the recollection of the distance of a foot or an inch.

Write out the results as follows:

Length estimated by inspection, - -	7 ft. 0 in.
Breadth „ „ - -	4 ft. 6 in.
Actual length of bench top, - -	8 ft. 0 in.
Actual breadth „ - -	4 ft. 0 in.

3. Measure carefully in inches all the dimensions of the given box (use a box of weights for example), and write out as follows:

Length of box, - - - - -	$=6$ inches.
Breadth „ - - - - -	$=2\frac{5}{8}$ „
Depth „ - - - - -	$=1\frac{3}{4}$ „

4. Make two marks upon your book so as to be, in your estimation, 6 inches apart. Correct the distances by your scale.

When out of doors make a calculation of the number of paces between two fixed points. Step the distance afterwards.

5. Guess the diameters of a penny and a halfpenny. Measure them accurately afterwards.

The diameter is the longest straight line which can be measured on the given body.

State the answer as follows:

Diameter of penny, - - - -	$=x$ inches.
Diameter of halfpenny, - - - -	$=y$ „

6. Take a given length, say that of a pen or book, estimate a distance equal to twelve times the selected length, and correct your distance by measurement with the pen or book.

The selected length is a standard for this occasion, and remember that the yard is fixed upon for a standard merely because it is a convenient length.

7. Compare two lengths; that is, find out how many times each length contains a given standard.[1]

The numbers will indicate the *relation* existing between the lengths. We can say, for example, that they are as 3 to 4, or that the lengths are related to each other just as are the numbers 3 and 4. The process is shown in the diagram below.

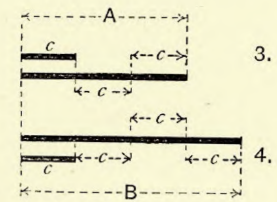


Fig. 2.—The process of comparing two lengths by means of a standard. *A* and *B* are the lengths to be compared. The standard *C* is applied to each length in turn, and it is seen to be contained three times in *A*, and four times in *B*. These lengths are therefore related to each other as the numbers 3 and 4 are related.

8. Measure the above lengths again by using a different standard, and satisfy yourself that the change of standard does not affect their *relation to one another*.

The lengths themselves are not changed, and their relation is the same; but the standard is altered, and *different numbers are obtained* when we use it in measurement.

9. Given two lengths, find out on the foot-rule a suitable standard for comparing them.[2]

The answer must be written as follows:

The lengths given were compared by means of . . . . as a standard, and were found to be in the relation of x to y (x and y standing for numbers).

10. Given two lengths, find out their relation without using a scale, but make a standard for yourself. A suitable length of string or a strip of paper will serve.

A little thought will bring out the value of this experiment in adding to the meaning of a standard, and will also show the

[1] Suitable lengths should be selected, so that a standard of an inch will be contained a whole number of times in each.

[2] Small commensurate lengths should be given.

connection of the process with the arithmetical problem of finding the Greatest Common Measure.

The relation will be expressed by two numbers.

An exact resemblance to the arithmetical method of finding the Greatest Common Measure may be seen in the following experiment:—Find out how many times the one length wholly contains the other. Mark on a strip of paper a length equal to the remainder. Ascertain whether this length is contained in the smaller of the original lengths any given number of times without leaving a remainder. If a remainder be left, see if this is contained in the first remainder a whole number of times. If it be contained without a remainder, then this length will be found to be a *common measure* of the two lengths, and a convenient standard by which to measure and compare the lengths given. Moreover it will be the *longest* standard available. This exercise is represented in the diagram below.

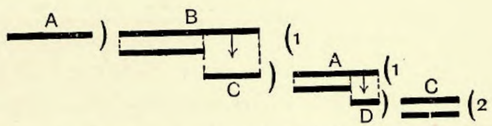


Fig. 3.—The diagram shows that the longest standard by which two lengths may be measured, can be found out by an operation, which is the same as that used in finding the Greatest Common Measure of two numbers.

A and *B* are the two lengths, and *D* is the longest standard which measures both without leaving a remainder.

A is contained in *B*, with *C* remaining.
C ,, *A*, ,, *D* ,,
D ,, *C*, ,, nothing remaining.

We now need to prove that *D* is contained in both *A* and *B* without a remainder.

Since *D* is contained in *C* without remainder,
then *D* is also ,, *A* ,,
(*for D and C together make up A*).
Since *D* is contained in *A* and *C* without remainder,
then *D* is also ,, *B* without remainder
(*for A, or a multiple of A, together with C make up B*).

11. Find out the distance between a given point and another which is inaccessible from it, that is, measure a distance alongside which the standard cannot be laid.

In order to perform this, some indirect method must be followed, and various examples should be suggested. For the present it will be enough to make use of several straight-edges[1] which carry the points, so to speak, out to such a position as will enable their distance apart to be measured. We may say that the straight-edges give us a distance equal to the inaccessible one, provided we can be sure, from our eyesight and judgment, that the straight-edges, which touch the two given points, are equally distant from one another at all points, or, in other words, that they are parallel. (See Fig. 4.)

[1] Any rod, bar, or lath with a straight edge.

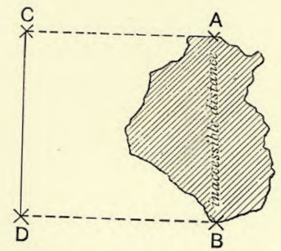


Fig. 4.—Diagram showing how an inaccessible distance *AB* may be measured by means of another distance, *CD*, which has been made equal to it.

12. Draw a straight line on paper, and lay off on it the following lengths in succession: $\frac{3}{4}$ in., $1\frac{1}{4}$ in., $\frac{1}{4}$ in., $2\frac{1}{2}$ in., $3\frac{1}{8}$ in., and $1\frac{1}{8}$ in. Then find out by means of the scale the total length thus obtained. Also write out the lengths expressing the fractions in eighths, and then add them up, showing that the total length is 6 in. and 16 eighths, or 8 in. in all.

13. Find out, by division of a line one inch long into four equal divisions, and also into twenty equal divisions, how many twentieths of an inch are contained in three-quarters of an inch. How many hundredths would there be? Express this as a decimal.

14. What is the longest length which is contained without remainder in both 20 and 32 inches? Write out an accurate description of how you obtain your answer.

15. Measure off along a straight line in succession 1·2, 1·3, 1·4, 1·5, and 1·6 of an inch, dividing an inch into ten equal parts for the purpose. Ascertain the total length of the line by actual measurement, and also by addition of the numbers themselves.

16. Measure off along a straight line the following fractions of 10 inches :—11, 12, 13, and 14 hundredths. Measure the total length, and then express the lengths in decimals of one inch, and add together.

SUMMARY OF METHOD OF MEASURING LENGTH.

12. In the exercises carried out we have learnt that in order to make a measurement of length, **the first requirement is some length by which to measure.**

We may or may not need to know a length accurately. The degree of accuracy should depend upon the purpose in view. We do not need the dimensions of a room to a very small fraction of an inch. But we may want to know the thickness of a wire with the greatest accuracy.

The standard length, or length used for measuring others, is a solid body, of wood, steel, tape, etc., upon which the standard length is marked.

Any length may serve as a standard. The length used should depend on the length to be measured. The distance between two towns may be given in miles, the length of a box may be measured in inches.

For the sake of convenience the *instrument actually used in measurement* may contain the standard many times, and may show divisions or fractions of the standard. A carpenter's rule is generally a piece of wood, two feet long, divided into inches and fractions of inches.

13. The process of measurement consists in finding out how many times the standard length is contained in that being measured. To find out the number of times, the standard is laid alongside the given

length so that two ends may lie together. A mark is made on the given length to agree with the further end of the standard, and the standard is again laid alongside the length, but this time with the mark just made serving as the starting-point. This process is repeated until the whole length has been traversed. **We depend throughout on the sense of sight.**

The description of the process just given takes for granted that the distance we wish to measure is the *shortest distance* between two points, and also that there is nothing to interfere with laying off the standard in the manner described. It is important to understand quite clearly that it has been *assumed* in all the previous exercises and observations that all lengths have been measured *in a straight line.* It is not easy to give an exact and full description of what is meant by straight, but it will be enough for our present purposes to say that *straight* means *even,* and a straight line is one which lies evenly between its extremities. A straight line going from one point to another lies evenly between those points. When we measure the distance between two points, we measure along this straight line, for when we speak of distance we mean *the distance measured along the shortest way,* and along a straight line is the shortest way.

14. If it be found out that the standard is not contained an exact number of times in the length measured, that is, if we find at the end that there is a length remaining which is less than the standard, then this may be expressed as a **fraction of the standard.** In order to find out what fraction remains, we continue the process, but use the small length as a standard, seeing how many times it is contained in the original standard. If it is contained 4 times it is a $\frac{1}{4}$, if 10 times $\frac{1}{10}$, etc., of the original standard.

A diagram will serve to make this still clearer. *A*, Fig. 5, represents the ordinary process of measurement, the smaller length serving as standard. The standard is contained 3 times. In *B* there is a remainder. How is it related to the standard? This is shown in *C*. It is found, by using it as a standard, that it is contained 3 times in the original standard. The length shown in *B* is therefore $3\frac{1}{3}$ times the standard. We have now carried on an operation which is the same as division in arithmetic, but instead of *numbers* we have used *lengths*.

FIG. 5.—A diagram showing that a fraction is obtained by measuring the standard itself by means of the remainder.

In *A* is seen an example of measurement in which no remainder is found.

In *B* the measurement shows a remainder (that is, a length smaller than the standard).

In *C* is seen the old standard being measured by the remainder as a new standard. The remainder found in *B* is therefore said to be one-third of the standard.

15. The common English standards used in measurement are the inch, foot, yard, and mile. These standards are related in a simple way to one another as follows:

12 inches = 1 foot.
3 feet = 1 yard.
1760 yards = 1 mile.

The convenience of the different names is great, for

while it would be *possible* to use only one standard and one name, it would be *inconvenient* to use large numbers, say the number of inches from London to Tonbridge, or to use small fractions, say the fraction of a mile which is equal to the length of the table.

It must not be forgotten that when we use a fraction of a standard, or when we say a length is $\frac{1}{16}$ of an inch, we mean to say that the given length is contained 16 times in an inch.

16. It is perhaps not necessary to point out that the terms *breadth, depth, height,* and others, all refer to lengths. They are all distances or lengths, which have received different names in order to describe difference of direction. In other words, **various distances may be measured on the same body, and to these distances various names are given.**

Lastly, it must be noted, and never forgotten, that a length when expressed in words or writing must consist of **a number and the name of the standard used.** We say a length is 10 feet, 6 yards, 3 miles, and so on. Here we have the two necessary portions of the description—1st, a number; 2ndly, a name. The dimensions of bodies and distances generally must always be given in this form, namely, *so many times the standard*, and then we express all that can be expressed about the given quantity. It is always permissible to shorten the name of the standard, as 5 ft., 3 yds.

EXERCISE.—Write out *in your own words* a description of the method of measuring a length. Describe the instrument used, the process of measuring itself, and lastly, the manner of stating the result of the process.

SOME INDIRECT OBSERVATIONS.

17. In the preceding measurements it has been taken for granted that the bodies measured and the standard used have not changed during observation. In other words, they are all *constant*. Measurements could not be made if they were not constant. All our measurements for the present are of things which are fixed. The measurement of bodies undergoing change, and of change itself, forms a special branch of study.

On the other hand, the measurements have taught us more than the exact dimensions of the bodies used. We have learnt that bodies entirely unlike in some respects may have one property or one feature in common. Therefore, in measuring the dimensions of bodies no attention is paid to the kind of body; nothing but distance is noted. And this is a very easy beginning of an important discipline,[1] **to be able to pay attention to one property of a body at a time.**

It often happens that a given property cannot easily be disentangled from others, and our perception or our training is not always good enough to enable us to observe it rightly. It was not likely that we should find difficulty in exercises such as those selected; there was no fear of confusing one property with another. But this was only because the dimensions of a body are, as a rule, by far the *most prominent* of its properties; and, as a rule, by far the most constant. It may also be added that they are selected for the first lesson because they are the simplest properties, and *the easiest to measure.*

18. It is also clear that **we can only measure between fixed points.** There must be something which does not

[1] Discipline, *training*, *instruction*.

change with regard to its surroundings, or measurements are impossible. It is not difficult to understand that several objects may be moving together and yet *preserving their distances from one another*, *e.g.*, the objects in a moving railway carriage. Distances may remain constant in a rapidly moving railway carriage, just as they may on the surface of the earth, which is rapidly moving. It is also possible that other changes of property may be taking place, and yet the property we are observing may be constant.

EXERCISE.—Write out all that you may have observed during the measurement of length, limiting yourself to the bodies measured and to the means of measurement. For example:—What did you measure with? How did you measure? Were the bodies which you measured alike? In what did they differ? Could you measure if matter did not exist?

MEASUREMENT OF DISTANCE BETWEEN TWO POINTS NOT CONNECTED BY MATTER.

19. The preceding section has omitted, for the sake of clearness, all mention of the distinction between *space* and *matter*. It is clear there is a difference between solid, resisting, matter and space, which is unresisting and apparently empty. Matter can be taken and handled, but space cannot. And it is extremely important to remember **that distance has the same meaning whether it is from one point in space to another point, or from one point or extremity of a body to another point or extremity on it.**

The operation of measurement is the same in both instances. In measuring the length of a box we may say that we are measuring the distance between two points or positions in space, which can be looked on as coinciding[1] with the extremities of the box, or we may say that we are finding out the distance between two points on a solid body. The statements are equally true.

EXERCISES.

1. Ascertain the distance between fixed points upon two benches. Lay a straight-edged piece of wood, or stretch a string across, in order to be certain that you are measuring the *shortest* distance.

2. Measure the distance between a mark on the wall and another mark on the floor. Measure also the distance of the two marks from the line made by the wall with the floor.

3. Extend the last measurement by marking on the floor several points at equal intervals from the wall; then measure the distance of each from the same mark on the wall, and write out the results as follows :

Distance of wall mark from floor-line, - - 2 ft. 6 in.
" of 1st floor-mark from same, - - 2 ft. 6 in.
" between each consecutive floor-mark, 6 in.

a in. =	length of straight line joining wall mark with 1st floor-mark.				
b in. =	"	"	"	2nd	"
c in. =	"	"	"	3rd	"
d in. =	"	"	"	4th	"
e in. =	"	"	"	5th	"
f in. =	"	"	"	6th	"
g in. =	"	"	"	7th	"

It may be observed from the above exercise, that whereas the distances between the marks on the floor are alike, the distances from them to a given fixed point on the wall do not differ by equal lengths. Some thoughts about the meaning of an angle should have occurred to the mind. This and other subjects, such as direction in space, will be treated at a later stage.

[1] To coincide is to fall together, or to occupy the same position.

4. Measure the height of the top of the bench from the floor, by taking a straight lath, sufficiently long, and marking on it the distance to be measured. Then measure this distance on the lath in the ordinary way.

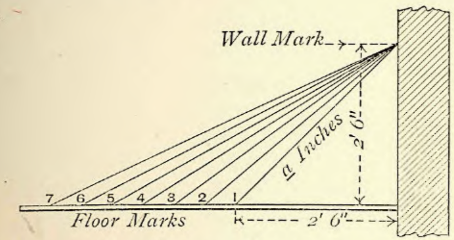


Fig. 6.—An example of measurement between points not connected by matter.

AIM OF EXERCISES.

The plan adopted in these exercises is an example of a method frequently used in measurement. It is not always convenient nor possible to measure a distance directly. In such case we may find out, or make, some distance equal to the one to be measured, and more capable of being laid alongside the standard for the purpose of comparison.

Close attention should be paid to the fact that you cannot measure a distance unless it be marked by fixed and visible objects. This has been already pointed out. In the examples of measurement just taken, when distances between points not connected by matter have been ascertained, there were, nevertheless, material points needed to mark out the extremities of the distance, otherwise no measurements could have been taken. Moreover, in these cases, it has been found necessary to interpose some solid in addition to act as a guide for measurement.

It matters little whether we are measuring accurately or roughly, whether the length to be measured is on the surface of matter or through space, but one fact should be by this time clearly grasped **that no measurement of length can be made except from one portion of matter to another.** This is nothing more than saying that there must be a starting point and an end to the measurement.

In the diagram below we can perceive that *BC* is a definite measurable length, because of the solids *AB*

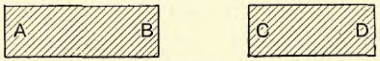


FIG. 7.

and *CD;* but take away *CD*, and there is nothing to measure unless it be some distance marked on *AB* itself.

CHAPTER III.

MEASUREMENT OF MASS.

DESCRIPTION OF METHOD EMPLOYED.

20. The word **mass** is used to denote **quantity of matter.** The meaning of the word quantity has been shown. It answers to the question *how much*, or *how many times.*

As to the word matter, we must be content for the present to let it stand for all the various objects which surround us, and are made known to us by the senses. Houses, trees, stones, water, cloth, wood, are a few examples of the great variety of matter which helps to make up nature.

Anything which we can touch or see is matter in some form or other, and the quantity of matter in any body is called for shortness its *mass.*

It is important to learn, that however varied may be the appearances and properties of the objects we perceive, they are alike in one respect, they are all specimens of *matter.*

To make all our ideas about matter quite correct is far from easy, but it is not difficult to be correct as far as we go; and, *provided we use the right methods,* it is not difficult to gain accurate knowledge about matter. Now the first step in trying to understand about matter is taken, when we begin to measure its *quantity*.

21. The first exercise, in measuring quantity of matter or mass, is to find out **when two masses are equal,** that is, when two bodies contain an equal quantity of matter.

This was easy enough in the case of length, because it was only necessary to place the two bodies, upon which the lengths are marked, side by side; and the eye can judge whether they are equal.

And if this be not possible, through the bodies not being movable, or through the points not being joined by matter, it is easy to test the equality by means of some third length. **If two lengths be equal to the same length they are equal to one another.**

It is true a similar statement may be made about masses. **If two masses be equal to the same mass, they are equal to one another.** But it is not easy to tell *when two masses are equal.* Our eyesight does not inform us in the way in which it informs us about length.

In the case of mass, the sense of *touch* must be used, if we are to estimate it directly. This sense is seldom to be relied on in the same way as sight can be. What is experienced is a feeling of resistance, and matter is always capable, in some degree, of causing this feeling, but it gives us *less definite information* than sight.

22. As an example of the difficulty in estimating mass directly, try to make two pieces of wood equal as to their quantity of matter. Use a knife to cut away from one or the other if necessary, and estimate by holding them out in the hand. Constant exercise of this kind would make the sense more accurate, but not accurate enough for scientific purposes. Letter-sorters are very skilled in estimating the quantity of matter in letters, but in case of doubt they use an instrument.

Now we have to learn the use of instruments for measuring mass, even before we can understand the *reasoning* or *principle* upon which they are constructed. This is not an uncommon experience. An engine-driver does not necessarily know much about engines, but he makes the engine do what is expected of it, and many other similar instances could be given. If it were not so, much of the activity of the world would be checked; and certainly our progress in science would be delayed considerably, if it were necessary to understand the principle of the balance before proceeding to use it.

DETERMINATION OF EQUAL MASSES.

23. In order to find out when two masses are equal we try or test them by observing if they may **act alike under the same circumstances.** The method here described will serve as an example of such tests of equality of mass.

Attach, by one end, about 6 in. of india-rubber cord (of any diameter from $\frac{1}{4}$ to $\frac{1}{8}$ of an inch) to a firm support, by means of string tightly tied to the india-rubber cord. Tie firmly to the other end any pan or small plate which will serve to hold small objects. Support behind the cord a lath, upon which marks may be made. Fasten a thin piece of wire at the lower end of the cord, so that it may serve as a pointer to show on the lath any alteration in the length of the cord. Mark the lath with a pencil so as to show the present position of the pointer.

Now place any suitable body, such as a piece of wood, on the pan; and mark the lath again to show the alteration in length of the cord. A mass sufficient to make this alteration quite distinct should be used.

Remove the body and note that the pointer returns to its *original position.*

Replace the body on the pan. Observe that the same change in the length of the cord is produced; and that when the body is again removed the cord returns to its original length.

This observation may be repeated several times.

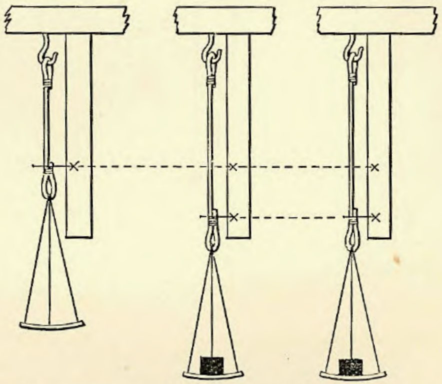


FIG. 8.—The *same* mass produces an equal change in the same body on each occasion.

24. We have now observed that the same mass has, *on each occasion*, produced the same change in the india-rubber cord. We have been able to make the observation by reason of the india-rubber cord resum-

ing its original length when the body is removed. If the change were permanent we could not make use of the substance. It would not enable us to say that like masses always produce like changes, or as we may also express it—**the same mass always acts in the same way.**

It is not difficult to anticipate the next stage in our observation. Procure another piece of wood and adjust its mass, by cutting away with a knife, until it produces in the length of the cord the same change as the other body. Make sure by several trials that the cord is equally stretched by both bodies.

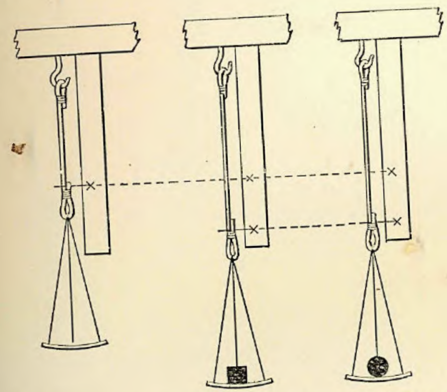


FIG. 9.—*Equal* masses produce equal changes in the same body on each occasion.

These two bodies will then be equal in mass. **They will contain the same quantity of matter.** The ground for this statement is that they have acted alike under the same circumstances. There could scarcely be a method of reasoning more convincing than this, in spite of the fact that the roughness of the material and apparatus may cause bodies to appear equal in mass when they are only nearly equal.

OTHER MEANS OF DETERMINING WHEN MASSES ARE EQUAL.

25. The method of determining equal masses by means of an india-rubber cord is one example of numbers of the same kind. It has been selecte first on account of its simplicity.

For the india-rubber cord we may substitute spiral of brass or copper wire,[1] and carry on th

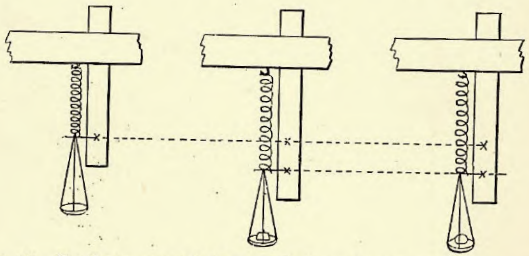


Fig. 10.—Further means of showing that equal masses produce equal changes in the same body.

experiment in the same manner. The lower en of the spiral should have a pan and a pointe attached to it. The stretching of the spiral is n quite so easy to mark as the stretching of the india rubber cord, owing to irregular movements; but goo results may be obtained with care.

[1] The spiral may be obtained by winding thin wire with the aid of a lathe. It is rolled on a cylinder, inserted in the lath and turned by it. The larger the diameter the more easily i the spiral stretched. The wire should be held tightly whil being wound by the lathe.

26. Another method, which will give good results, is obtained by observing when the same effect is produced by two masses in an instrument, such as that described below (Fig. 11):—A board may be cut to the shape *A B*, and a pin passing through it at *C* will enable it to be suspended. Then the mass to be measured is suspended from *A*. The exact position at which the board will rest, when a mass is placed at *A*, may be shown by a small plumb-line fixed at *C*. Marks drawn upon the board, as shown, will serve to denote the position in which the board comes to rest. Equal masses will cause it to take up the same position with regard to the cord which hangs from it.

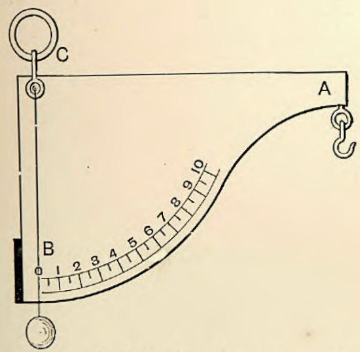


Fig. 11.—Another mode of showing when two masses are equal.

Other instruments may be used for the purpose of showing when masses are equal, and a variety are in use, but the principle is the same in all. It is only needful to devise some plan **by which the same change may be brought about as often as it is required.**

27. It may be noted at this point, that these equal masses with which we have lately been dealing are, as far as we know, **equal only with respect to the tests applied to them.** It is true that they *may be* equal in other respects, and we shall learn later on that they are equal in a very wide sense; but it must not be supposed that equal quantities of lead and wood will act alike on all occasions. Lead, for example, does not burn like wood. It does not float in water like wood. This is the same thing as saying that they have some properties which are obviously unlike, and that they are, in other words, **different kinds of matter.** But, as we have seen, different kinds of matter may be equally *pulled or attracted to the earth,* and then we may say they are equal in mass.

The limits, therefore, to the meaning of the word equal, when applied to matter, must be remembered. **Equal masses are necessarily equal only in the sense that they are equally pulled towards the earth,** and hence they are equal only in the changes they produce in such experiments as have just been performed. In all other properties there may be great inequality.

EXTENSION OF THE METHOD TO OBTAIN MULTIPLES OF A GIVEN MASS.

28. In the previous sections we have been entirely occupied with ascertaining when two masses are equal. Various tests of this equality, all of the same character, have been given. But if we can procure two equal masses, we can obtain three, four, five, and any number of equal masses. That is, we can obtain *any multiple* of a given mass.

It is easy to perceive, **that we should possess a general scheme of measurement of mass had we any standard common to everyone.** That is, we need some definite quantity of matter which can have a name given to it. Other bodies can be procured containing an *equal* quantity of matter, and the *same name* is given to them. Such a standard exists, and is called a *Pound.* This is generally used in England and most English-speaking countries. There are innumerable masses of one pound throughout the country, which can be referred to at any time. Since we can easily determine when two masses are equal, it is easy to copy the standard indefinitely. The original standard is made of platinum, and is preserved in London. One-sixteenth of this mass is called an ounce.

29. But it is also easy to perceive, that with such instruments as have been described, the process of weighing, or finding out the mass of a body, would be rather tedious, if we could only use our instruments to find out when masses are equal. **We may extend their usefulness by finding out, and marking, the different changes which are produced in them by various multiples of the mass originally taken.** In other words, we can find out what change is produced, in any suitable body or instrument, by masses equal to one, two, three, four, etc., times the standard. The instrument may be marked to show, for example, one, two, three, four, etc., pounds. It is probable that the limit of change which can be shown is soon reached, but some definite extension of the operation will be possible. It may not be possible to show more than from one to ten pounds on the same instrument, for reasons which will vary according to the instrument.

It will, of course, be found, that in the instruments which have been made for general use, the standard selected is the pound, and the instrument is *graduated* in pounds, or fractions of a pound. This *graduation* may be described as a process of marking on a scale certain positions, and expressing the values corresponding to them, so that when the pointer is at one of these positions, **we may know from our original observations the quantity which brings it there.**

Some of the instruments should now be practically graduated, or an important lesson will be lost; and, in addition, the graduations already made on some instruments should be carefully tested.

In the first place, an ordinary spring balance, which is supported on a hook, should be tested, by adding in

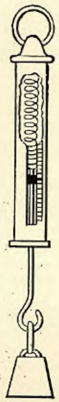


FIG. 12.—An ordinary spring enclosed in a tube, which is graduated so as to show the extent to which the spring is lengthened. Known masses have been used in obtaining the various marks.

succession the several masses which are marked on its scale. As each mass is added, the pointer should stop at its corresponding mark. It is possible that slight inaccuracies may be detected. Whether or not the spring returns to the same position of rest after being stretched should be noticed (Fig. 12).

Afterwards graduated scales should be constructed for the springs and india-rubber cords which have been

used for obtaining equal masses. This may be done roughly by supporting either cord or spring by a pin or screw fixed in a lath upon which paper has been pasted. The cord or spring must not touch the lath when it is held upright. Points should be marked on the paper in the first place, in accordance with the observations, and then afterwards lines may be ruled through the points.

Efforts should now be made to design simple instruments for weighing, and the results obtained from their use should be compared with each other. Various methods in which the bending, stretching, or twisting of material is used may be suggested, and the different ways of balancing or supporting bodies at one or more points will give results.

EXERCISE.—Write out in your own words a description of the method employed in measuring mass. Give a summary of what you have read under the heading of Measurement of Mass. Compare the method with that used in measuring length. State what is done in finding out when two masses are equal, how masses are compared with one another, and the method employed in the ordinary operation of weighing.

ANOTHER INSTRUMENT FOR MEASURING MASS.

30. An instrument[1] which affords very convenient means of measuring mass is illustrated in the diagram

[1] This instrument is very cheap and very useful for class purposes. They may be obtained graduated in grams, and they prove convenient in rapid weighing when great accuracy is not essential. You may observe that they give readings which are not always accurate, by noting the difference made when a body is placed at the edge of the pan.

below (Fig. 13). The object to be weighed is placed on the pan *A*. The curved portion *BC*, which has considerable mass *B* attached at its lower end, is pushed to one side by the depression of the pan.

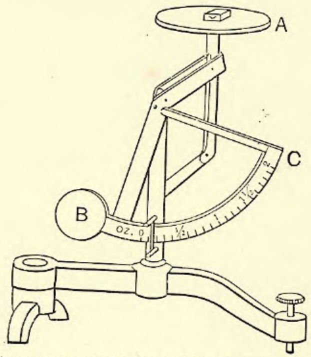


FIG. 13.—An instrument which enables masses to be compared with one another and with a standard mass by showing the distance through which the mass *B* is raised.

The larger the mass of the object placed on the pa[n] the further is the portion *BC* moved to the left. [It] will be seen that the mass *B* is raised according as the pan is lowered, and hence we have the mass on *A* balancing in a certain fashion the mass *B*. The curved portion *BC* is graduated to display how far it passes a bar, which serves as an index or pointer.

The instrument having been tested with known masses, the values corresponding with various positions of the curved scale are marked thereon.

EXERCISES.

1. Test the correctness of the graduations of this instrument by placing on the pan in succession the various masses which are denoted on the scale.

2. Find out the mass of the given body without using the scale. The scale may be temporarily covered by pasting paper over it. Mark the position of the curved portion when the object is placed on the pan, and ascertain the mass (using "weights") required to bring the instrument again to the same position. "Equal masses produce the same effects under the same conditions."

3. Graduate the paper which has been pasted over the scale for the last exercise.

4. Fix a scale behind the pan of the instrument so that you may measure the distance through which the pan is depressed by the addition of a given mass. Draw up a table showing the distance for each addition of 2 grams.

5. Make a drawing of the whole of that portion of the instrument which moves when a body is being weighed, and show how this movable portion is suspended from the stand. Compare the method of weighing by this instrument with the appliance in Fig. 11, and note that they are alike in principle.

6. Note that equal lengths on the graduated scale correspond with equal masses, and that this instrument is one which *measures masses as if they were lengths.*

7. Devise a method of measuring, by means of two balances, a larger mass than is possible to be weighed when one balance alone is used. Bring two balances together and place a short bar across both pans, so that each may aid in supporting the body, which is placed midway on the bar. Make your own observations.

THE USE OF THE BALANCE.

31. We now come to avail ourselves of an instrument called a *balance*, which is simple enough in construction, but not so simple in the principles which underlie this construction. Hence it will be as well to leave to a later stage the explanation of these principles. Much will be learnt by carefully taking the balance to pieces, and observing the construction of each part.

A good form for use in the laboratory is shown in Fig. 14.

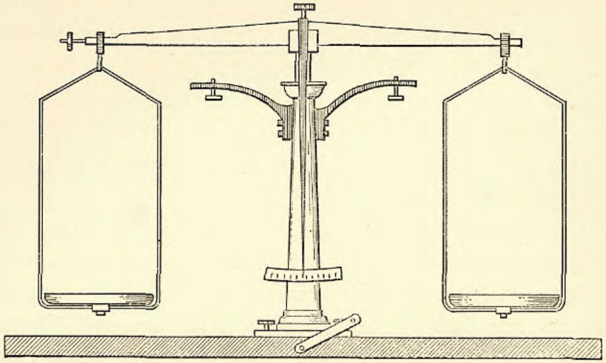


FIG. 14.—A form of balance suitable for laboratory use.

1. Place two pieces of wood on the pans of a balance,[1] and cut away from one or the other until the balance shows them to be equal.

Notice that we have to trust to the balance;

[1] Before any use of the balance is made by the class, it is advisable for the master to take one to pieces and point out its construction, dwelling especially upon the reasons which make it necessary that sensitive balances should be used with great care. The method of using it should be illustrated, and among other cautions the following should not fail to be given :—

1. Nothing should be placed on or removed from the pans while the balance is free to swing on the knife ; edges are quickly worn.

2. Only comparatively small masses should be measured, or the beam becomes strained and the knife edges become blunt.

3. The beam should be only slightly raised at first for the purpose of ascertaining whether the objects in the pans counterpoise. It is not until they appear to be about equal that the beam should be allowed to swing freely.

we have so far no means of testing whether the balance gives a true indication. It has been constructed for that purpose, and is certain to give a better result than you would get from your own sense of mass; but it is important to remember that we depend entirely in this exercise on the correctness of the balance. The balance may give correct results, but on the other hand, as far as we know, *they may be far from correct.*

The same kind of test is applied in using the balance as in using our hands, that is, the extent to which a body is *pulled downwards* is felt by ourselves and by the balance too; but in the balance we can make *two pulls oppose one another;* and when they are equal the balance swings evenly, or stands at rest with the beam horizontal, provided the balance be a good one.

2. By the same means make a third piece of wood equal to either of the other pieces. If the balance be an accurate one, we shall then have three equal quantities.

With these three masses we may now test the accuracy of the balance. We can ascertain if they are really equal, as the balance represents them to be.

3. Mark with a pencil the three pieces of wood supposed to be equal. Place No. 1 and No. 2 on the pans, and observe whether the b lance represents them to be equal. (If a good balance with an index and scale be used, this will be simple. The index will show equality by swinging an equal distance on each side of the scale, or by coming to rest at the centre of the scale.) **The next step is to reverse the places of the bodies. Put No. 1 in the place of No. 2, and No. 2 in the place of No. 1.** Note carefully whether the indication is the same in each case, and if you are uncertain, repeat the operation.

It is likely that the readings will not be quite the same, but they may be so nearly alike that care is needed to observe any difference.

4. The next step is to put No. 3 in the place of No. 1, and note the reading of the balance, and then in the place of No. 2, restoring No. 1. If the balance be correct the two readings will be *exactly alike.* In fact if the balance be correct, and if the three masses be really equal, then it makes no difference in what pans the separate masses are placed, or in what order they are taken: No. 1 with No. 2, or No. 1 with No. 3, or No. 2 with No. 3, *there will always be exact counterpoise.*

5. We now come to an exercise which will show us how we can be independent of the ordinary defects of a balance, that is, how we can be sure of getting two equal quantities of matter by the use of any balance.[1]

Place Nos. 1 and 2 in the pans, and cut away, if necessary, to make an exact balance. Then put No. 3 in the place of No. 2, leaving No. 1 untouched, and cut away No. 3, if necessary, to make an exact balance.

We have now put No. 3 in the place of No. 2, and ascertained that under the same circumstances Nos. 2 and 3 have *behaved exactly alike.* Since the circumstances are what they are, we shall be right in speaking of them as **equal quantities of matter.** *We are not able, however, to state that they are equal to No. 1.*

[1] It is advisable to make the balance read inaccurately before starting, either by altering the adjustment or adding a mass to one of the pans, placing it by preference underneath the pan.

6. Consider what would be necessary, to judge from the last exercise, in order to make all three bodies quite equal in mass; and then carry out your plan.

Notice that we can in this way obtain a mass equal to any number of times a given mass, or, as it is called, any *multiple* of a given mass. We may also perceive that the processes of *addition* and *subtraction* can be carried on with *matter* just as with *numbers.*

7. In order to show the independence of **quantity** and **kind** of matter, prepare, by methods similar to those described above, two equal quantities of different kinds of matter, for example, pieces of lead and wood. A number of equal quantities of these may be prepared.

However unlike in appearance lead and wood may be, it is quite correct to speak of a piece of lead as containing the same quantity of matter as a piece of wood. It is unnecessary to point out that the *sizes* are not the same in this case.

8. Hammer the lead and cut the wood into pieces, and then show that they still possess the same quantity of matter.

Change of mass is not produced by change of shape.

9. Assuming your balance to be correct, and making use of the masses previously made, prepare two masses equal to twice the mass of the piece of lead, and one mass equal to five times its mass. It will be convenient to the purpose to use shot, placing it in watch glasses.

10. Satisfy yourself that you can, from the three masses last prepared, together with the piece of lead, make up masses which shall contain 2, 3, 4, 5, 6, 7, 8, 9, or 10 times the piece of lead itself.

11. State what masses would be required to measure any mass up to a hundred times the mass of the piece of lead.

Take notice that we have now made the piece of lead a *standard mass,* and that with such a standard we can **measure** masses and **compare** masses with one another.

12. Prepare, by using shot, a mass which is one-eighth the mass contained in a given body. (A $\frac{1}{2}$ lb. weight may be provided.) Weights are not to be used. (The result may be readily tested by an oz. weight.)

13. Two bodies are provided. Find out the number of times the mass of the one is contained in the mass of the other. This must be carried out without the use of the "weights." Make use of shot for the purpose. (Select the masses so that one is a multiple of the other.)

14. Two bodies are given, and it is required to know how they are related in mass. That is, you must compare their masses, and you must do so by means of the standard mass provided.

Another standard is then provided, and the masses are to be again compared by means of it. Note that the *same relation* is shown to exist when this standard is used, although the numbers themselves are larger or smaller as the case may be.

15. What is the largest mass contained a whole number of times in 88 and 297 grams?

16. Find a unit by which to compare two masses which are given. Use a method similar to that used in finding the G.C.M. of two lengths.

17. The 1 gram weight is missing from an ordinary set of weights; how would you prepare a mass equal to 1 gram by the help of the others and a balance?

SUMMARY OF METHOD.

32. In the preceding measurements we have been occupied in finding out the mass of various bodies, that is, the quantity of matter contained in them.

You will remember that in measuring the dimensions of bodies no attention was paid to the kind of body measured. Lengths and distances are independent of the kind of matter upon which they are measured.

In a similar manner the **quantity** of matter has nothing to do with the **kind** of matter. The two facts must be kept distinct, and we must learn to measure all matter by the same method, whether it be wooden, brass, or leaden matter.

To what these differences in appearance and property are due will be learnt later. At present the important lesson to be learnt is, that there is something which all kinds of matter have in common, that is *quantity* of matter, or **mass**. And we can find out this quantity by methods similar to those by which we measured lengths—that is, by selecting a **standard quantity.**

We have not measured with the *utmost* accuracy, any more than we measured lengths with the greatest attainable précision, but we have measured in the right manner.

33. In comparing one mass with another a standard must be used. We can then say, for example:—

A mass A contains the standard 5 times.
A mass B contains the standard 11 times.

These statements tell us that the masses of the bodies are in the same relation as the numbers 5 and 11. The same result will be obtained with any standard, though the numbers may now be 10 and 22, or 15 and 33, etc. This should be carefully attended to, and should be practically tested. The numbers will continue to be in the **same ratio.** 5 is related to 11 in the same manner as 10 is to 22, or 15 to 33.

Note that we have carried on operations, and applied tests, similar to those used in measuring and comparing lengths. The *standard mass* has played the same part as the *standard length,* and *masses* have been compared by the same processes as *lengths* were compared.

But there is one difference between the measurement of length and that of mass which is worth noticing. In measuring a length, it is the rule to ascertain directly how many times the standard is contained in it, by "laying-off" the standard along it. But the act of weighing with a balance consists of two parts, the first being to produce counterpoise by the required number of "weights," the second consisting in counting up the number of times the standard is contained in these "weights."

34. It should also be noted that in the *comparison* of masses any standard which will serve as a common measure—that is, any standard which is contained in each of the masses a *whole* number of times—will give us the required information, namely the relation between the masses as represented by numbers. But in order to convey to the minds of other people the *actual quantity of matter* we have found in a given body, we must make use of some standard of which the value is known to them, or to which they can refer.

Such a standard would be, among others, a pound or an ounce, for these quantities are copied and are ready to hand in all parts of the country. But it must be remembered that in saying that a given body contains 5 lbs. of matter, or, as it is more generally described, *weighs 5 lbs.*, we do nothing more than state that it has 5 times the quantity of matter contained in a certain body which everyone has agreed shall be called a pound. This body called a pound, or some other body equal to it in mass, has been *compared with* the body described as weighing 5 lbs.

35. EXPLANATION OF TERMS USED IN THE PRECEDING SECTION.

Observation.—The act of observing or using the senses in order to learn anything. To observe is to use one or more of the senses with the object of gaining knowledge.

Measurement.—The act of finding out how many times one quantity is contained in another of the same kind. In measuring we compare two quantities of the same kind.

Quantity.—That which can be measured, and can be expressed as so many times any other quantity.

Standard.—A quantity by which other quantities of the same kind are measured. The size or magnitude of the quantity selected as a standard depends upon the object of the measurement, but it must be a quantity which can be referred to by others besides those who have made the measurement. A yard is a standard in general use, but a piece of wood which may serve as a means of measurement on some special occasion conveys no meaning as regards length to other people.

Unit.—The standard in any system of measurement. The word *unit* refers to the starting-point of calculation. It is that quantity which is denoted by *one.*

Length.—Distance between two given points. When we think of distance, the idea of passing from one point to another comes to our mind.

Dimensions.—Such lengths measured on a body as give information as to its size.

Instrument.—An object which has been made for the purpose of observing or measuring accurately.

Fraction.—A portion of the whole, or, we may say any quantity which is less than a given quantity is a fraction of it. A decimal fraction states how many tenth parts (or hundredths or thousandths, etc.) of the whole are contained in the fraction.

Constant.—Standing still or fixed; unchanging.

Property.—That which belongs to, or is possessed by, a body. Dimensions, colour, shape, etc., are properties of bodies.

Multiple.—A multiple of a quantity or number contains that quantity or number an exact number of times.

LETTERS OR SYMBOLS, a, b, x, y, etc.—Are used to stand for numbers which have not yet been ascertained. x ft. means an unknown number of feet.

MATTER.—That which we can perceive by the senses. The objects constituting nature are forms of matter. It exists in and occupies space.

MASS.—The quantity of matter in a body.

WEIGHTS.—A set of bodies of which the masses are such multiples and fractions of the unit mass as will enable any required masses to be readily made up by groups of them.

POINT.—Any mark to indicate position.

METHOD.—The mode, plan, or scheme by which anything is performed.

BALANCE.—An instrument enabling us to tell when two masses are equal, and hence to measure how many times the unit mass is contained in the mass of any given body.

RULE OR SCALE.—A body upon which are marked multiples and fractions of the unit length for convenience in measuring.

VERTICAL.—A term describing the direction taken by a string at rest and supporting a body containing enough matter to straighten the string. A plumb-line shows the vertical direction. This is the same direction as the path of a body which falls to the earth from a position at rest above the surface of the earth.

HORIZONTAL.[1]—A term describing the direction taken by any line drawn on the surface of a liquid at rest.

[1] A horizontal and a vertical line form, when they meet, what is called a right angle. Further knowledge of these matters will be gained at a later stage.

CHAPTER IV.

GENERAL PRINCIPLES OF MEASUREMENT.

THE AIM OF MEASUREMENT.

36. It will be convenient to pause at this point, and make certain that we understand clearly what is meant by a **standard** or **unit**. We have observed that the first step in measurement is to know when two quantities are equal. The examples we have taken, length and mass, have been very simple in this respect.

Direct observation soon enables us to decide when two lengths or two masses are **equal**. In the case of lengths they are placed side by side, or referred to a third length. If possible, the marks showing the extremities of the lengths are made to coincide or come together.

In the case of masses we look to the balance, or other instrument or appliance used, and it may be here noted that we test equality of mass by the eyesight, just as much as we do lengths. Since we cannot trust the sense which directly affords us the feeling of mass, we have transferred **the test** from the sense of touch to that of sight. We *look at* the position of the pointer on the scale in order to ascertain if masses be equal.

37. The next stage in measurement has been to find out *how many times a standard quantity* needs to be taken to *make up a quantity equal to that being measured*, or as is said—how many times the standard is *contained in* the quantity under investigation.

It must always be remembered that sometimes the quantity is only a fraction of the standard, for the standard may be larger than the object measured.

All that we can assert *numerically* of any quantity is that it is so many times, or such a fraction of, another quantity of the same kind. We then give to it a number or *numerical value*, which of course depends upon the size of the selected standard quantity.

We never learn from direct observation anything more than equality or inequality of two quantities. It is advisable to give some thought to this statement.

We have always used the word *standard* to denote the *quantity selected as the starting point of measurement.* The word *unit* will be used afterwards when we begin to refer to certain standards which have been selected for special scientific reasons.

38. The quantities used for measuring have been either the familiar inch or ounce, or some quantity serving for that occasion alone. In the future, certain quantities called the **gramme** and **centimetre** will be used for most measurements of mass and length, and they will always be spoken of as **units.** These quantities will be used because they form part of a great comprehensive scheme of measurement which is universally adopted for scientific purposes.

It need not be pointed out again, that for measuring mass a unit of mass is needed, and for measuring length a unit of length. That is, **we measure by comparison with quantities of the same kind.** This statement seems scarcely needed with regard to measurements of mass and length, for no one would attempt to measure mass by means of units of length, nor to measure length by means of mass; but in the case of *more complex quantities* such mistakes are often made.

These measurements form a good illustration of what all measurements of quantity should be, and what they are in scientific work, namely, **the determination of quantity and its expression in terms of a selected quantity of the same kind by means of numbers, these numbers, whole or fractional, being called the numerical values of the quantities.**

THE MEANS OF MEASUREMENT.

39. The operations of measurement, as they have been carried on previously, would be *too slow for general use.* In order to bring the method and order of the process into prominence, each stage has been taken by itself. In the more rapid measurements which are generally used, several stages are taken together, and instruments such as *scales* or *rules* for lengths, and *boxes of weights* for masses, are used for the sake of more rapid reckoning. From those who use such appliances greater accuracy as well as more rapid work is expected.

The ordinary measure used by carpenters and mechanics, generally, is a *two-foot rule.* This will be seen on examination to be divided into inches, and eighths of inches, and often into smaller divisions; that is, each foot is divided into 12 equal parts called inches, and each inch is again divided into 8 equal parts, each of which is $\frac{1}{8}$ of an inch, and so on.

A careful inspection of a good scale will repay trouble. Make out the meaning of fractions by so doing; for example, in $\frac{1}{2}$ an inch there are $\frac{4}{8}$ (four separate eighths), in $\frac{3}{4}$ inch there are $\frac{12}{16}$ (twelve sixteenths). Now all these divisions are carefully denoted on the scale by numbers and lines, long and short (Fig. 15).

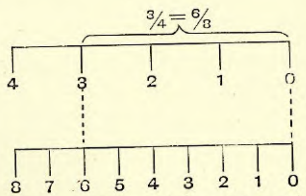


FIG. 15.—A diagram showing that $\frac{3}{4}$ and $\frac{6}{8}$ of the same length are equal. In *A* the divisions are *fourths*; in *B* they are *eighths*.

It must be remembered that every one of the scales is supposed to be taken from a certain standard yard deposited for safety in a box, carefully sealed up in a recess of a wall within the House of Parliament. From this standard a number of very accurate copies were made, and then these served in their turn as patterns to be copied. Hence there are a number of standards in existence, by means of which distance can be measured without referring to the original, which must be nevertheless regarded as the starting point of all measurement of length in this country.

40. In spite of the yard, with its subdivisions into feet and inches, being the standard taken for ordinary

measurements in this country, yet for most *exact* measurements, such as have to be made in the laboratory, the **metre** with its subdivisions is used.

The *metre*, which is really the basis of the **Metric system of measurement**, is used in all countries for scientific measurements, and in many countries for ordinary purposes. Compared with the yard, it is about one-eleventh longer; more accurately, it is 39·37 inches.

This length was chosen because it was supposed to be *one ten-millionth of a quarter of the earth's circumference*, passing through Paris,[1] and a bar of platinum was made in 1795 by Borda, in Paris, to represent this length and serve as the standard for all future measurements. Platinum was used, because that metal does not rust nor change from exposure to air.

Whenever a measurement is made in metres, a comparison is made with this standard metre, not directly of course, but by means of some of its numerous copies.

It may be mentioned here that this standard, as well as the yard, give their true value only when they are at a certain *temperature*, the yard at a temperature known as 62° Fahrenheit, the metre at the temperature of melting ice. *It is well known that the size of bodies alters with change of temperature.*

EXERCISE.—Write out carefully, without referring to the book, what you have learnt to be the principles of measurement, and what you know of the means of measurement. In your account, do not forget to point out—that we depend upon our eyesight in measuring

[1] The earth not being quite spherical, the circumference varies at different positions.

both mass and length, that the numerical value of a quantity is the object aimed at in measurement, that both whole numbers and fractions may constitute numerical values, that these vary according to the standard selected, that the standards used in a general scheme of measurement are called units, that the Kilogramme and Metre in Paris form the foundations of every gram and metre measure in existence, that they form, in fact, the starting-point.

41. A portion of a metre scale should now be taken in hand, and a copy of it made in your book. (The steel scales, which are about a foot long and divided into fractions of a metre and of the foot, are very convenient and instructive for use in class, Fig. 16.) On

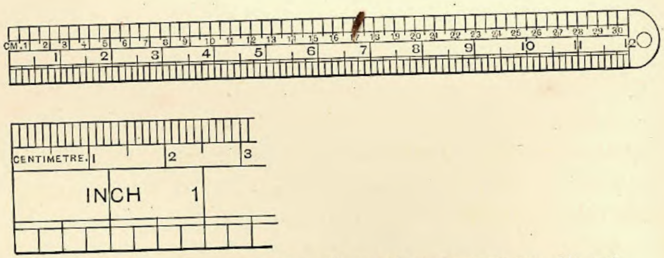


FIG. 16.—A portion of a scale with metric measure on one side and English measures of length on the other.

this scale the longest lines with figures against them mark the hundredths of a metre, or **centimetres**, and the smallest distances, of which there are ten in each centimetre, are **millimetres**.

Both these words will be frequently used afterwards, more frequently, indeed, than the word metre; for the metre itself, being too long, is not often used in measuring in the laboratory.

The fractions of the foot, being on the same face of the scale, may be compared with the divisions of the metre.

The full table of lengths will be:

1 metre = 10 decimetres = 100 centimetres = 1000 millimetres,
1 decimetre = 10 centimetres = 100 millimetres,
1 centimetre = 10 millimetres.

Or again:

1 millimetre = $\frac{1}{10}$ centimetre = $\frac{1}{100}$ decimetre = $\frac{1}{1000}$ metre, or
1 millimetre = ·1 centimetre = ·01 decimetre = ·001 metre.
1000 metres is called a kilometre.

Other names are given to multiples of the metre, namely, decametre for 10 metres, and hectometre for 100 metres, but they are seldom used.

It will be observed that the metre is subdivided always into *tenths*, that is, into *decimal parts*. A decimal is not written in the same way as an ordinary fraction, but in spite of this it is a fraction.

A line should be drawn, divided accurately into ten equal divisions, and one of these divisions again divided into ten equal parts, in order to illustrate the meaning and relative value of decimal fractions.

Arithmetic teaches us how to calculate with numbers and fractions, and it is useless to expect to make progress in practical science if there is any uncertainty about the ordinary rules of arithmetic, or about the meaning of fractions, for these are essential to measurement.

The following abbreviations may be used:

cm. for centimetre, and mm. for millimetre.

EXERCISES IN USE OF METRIC MEASURE: LENGTH.

N.B.—*A careful account of each experiment is to be written out in a note-book, and drawings are to be made wherever they can be used to illustrate what has been done.*

1. Ascertain the length of the given object, and express it in millimetres. Write out the length also in terms of centimetres, and also of a metre; that is, make use of the decimals required.

2. Measure the diameter of a penny, and express it as a fraction of a metre. Find the relation (or ratio) between the diameters of a penny and a halfpenny by measuring each in millimetres as carefully as possible.

3. Given a two-foot rule, measure the length of the room, and express the value in centimetres. A metre = 39·37 inches.

4. Given a metre, and a tape-measure graduated in inches, find out the number of inches in a metre by laying off a length of several metres, and measuring the length in inches. Be careful not to leave a blank space at the ends of the scale in laying off.

5. Ascertain the value of an inch in centimetres by observing the value of 10 inches, and dividing it by 10. Compare the result with that obtained when 20 or 30 inches are measured by a metre scale. State the number of millimetres in an inch. Note the increased accuracy obtained this way. A red blood-corpuscle is about $\frac{1}{3200}$ of an inch in diameter. How many would lie side by side in a length of 1 millimetre?

6. Draw neatly in your book a copy of a scale about 5 cm. in length, dividing it carefully into millimetres. Use drawing instruments, and persevere until a neat and accurate scale has been constructed.

7. A *chain* used in land-surveying contains 100 links, and is equal to 66 feet. Draw a line in your book equal to 1 link by use of the metre scale, and see that it be accurate to a millimetre at least.

8. Find out the circumference of a penny, by marking it with a spot of ink and carefully rolling the penny over paper, so that two marks from the ink may denote the length of the circumference. Find out the relation of this value to the diameter. Remember there is always the same relation for all circles.

9. Measure a given curved line drawn upon paper by finding the length of a piece of thread which will coincide with the curved line. Stretch the thread with the finger tips so as to coincide exactly with successive portions of the curved line. (The same curve may be given to the whole class, and the values obtained may be compared.)

10. Measure the circumference of a cylinder by using a piece of thread in the same way. Then make your result more accurate by winding the thread several times, counting the coils, round the cylinder, and then ascertaining the length of the thread. The coils must not overlie one another. The length divided by the number of coils now gives the value of the circumference much more accurately.

In fact, by this process we have taken the measurement several times, and have ascertained the mean value, in addition to which the chances of faults in handling are fewer.[1]

11. Find the diameter of a penny, cylinder, or roller, by placing its extreme points in contact with the upright faces of two blocks which have been placed upon, or by, a scale, so that the distance of the faces apart may be measured. The value of the reading depends entirely upon the blocks having true square edges (Fig. 17).

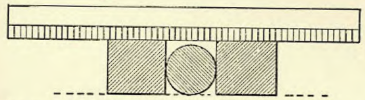


FIG. 17.—The use of callipers is explained by the arrangement of the blocks, which enable the body between them to be measured. (See Fig. 110 for callipers.)

The use of callipers as a substitute for blocks may be explained here.

There are many opportunities in this experiment of showing skill, and of learning how to avoid errors of observation.

[1] The same experiment may be performed with ribbon rollers and the strips of paper in which the ribbon has been rolled. These may be obtained from drapers.

The diameter of a shot or wire may be measured in the same way, and the result tested by the callipers. (Two scales placed on end on another scale will serve, if blocks are not at hand for this observation.)

12. Find the distance between two marks on two upright stands, for example, retort stands, or those used on an optical bench. The marks must be so placed that measurement is only possible by adding or subtracting lengths.

13. Find the difference in level between two points by measuring the distance of each from the floor. Extend this exercise by taking one of the points in such a position that its distance from the floor cannot be measured directly.

14. Find the internal depth of a watch glass as accurately as possible, then measure its total depth, and so calculate the thickness of the glass.

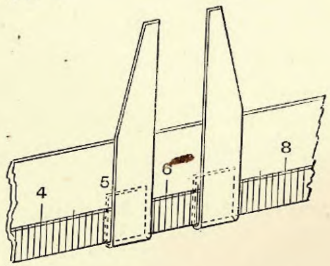


FIG. 18.—A rough pair of callipers formed by fitting pieces of sheet-brass on an ordinary scale.

15. Make measurements of the diameters of a cylinder and a sphere, by means of rough callipers made by adapting sliding pieces of brass to the ordinary steel measures (Fig. 18). The same measurements should be taken with ordinary callipers, by fitting them to the object, and then reading off the distance on a scale. Workmen's callipers should be used.

THE METRIC MEASUREMENT OF MASSES.

42. The word *mass* will be used throughout this book; but in consequence of the firm hold upon our

language of the word *weight*, there can be no serious harm in its use by the student for the present. The unit of mass is called the **gramme**. This word, which is of French origin, is shortened in English to *gram*.

The original standard, the starting point of all subsequent standards, is a piece of platinum adjusted by Borda and called a **kilogramme**. The mass which we call a gram is one-thousandth of the mass of this piece of platinum.[1] The gram is equal to about 15·432 grains, and beginners must not confuse it with a grain, nor with a drachm.

1 **kilogram** = 2·2054 lbs. avoirdupois. (The number 2·2 is near enough for ordinary purposes.)

A mass of $\frac{1}{10}$ or ·1 gram is called a **decigram**.
„ $\frac{1}{100}$ or ·01 „ „ **centigram**.
„ $\frac{1}{1000}$ or ·001 „ „ **milligram**.

The only multiple of the gram which is used is the kilogram.

43. For convenience in weighing, or finding the mass of a body, we use what are called sets of *weights*, that is, various pieces of brass and aluminium, carefully adjusted so as to contain convenient multiples and fractions of the quantity of matter contained in a gram.

An inspection of a box of these adjusted masses will show that they are arranged, so that any number of grams from 1 up to 100, or 200, or 500, as the case may be, can be made up by various groupings of the pieces of brass. And also any fraction of

[1] It is clearly safer to make the standard which is deposited for reference fairly large, so that any accidental alteration or change may be a small fraction of the whole.

the gram down to one-thousandth of the gram can be made up. A careful table of the masses in the box should be drawn up, and they should be carefully inspected so that they may be readily recognized. Much time is gained if this be done thoroughly well at the start. The table should be as follows, assuming 100 grams to be the largest mass :—
100, 50, 20, 10, 10, 5, 2, 2, 1, grams in brass;
·5, ·2, ·1, ·1, ·05, ·02, ·01, ·01, ·005, ·002, ·002, ·001, of a gram in aluminium or platinum (Fig. 19).

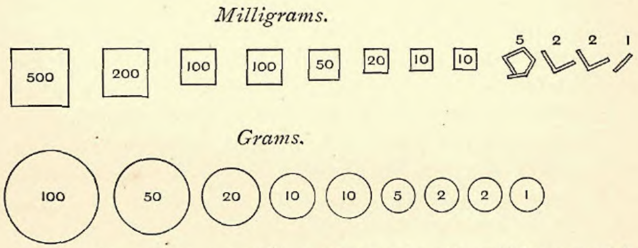


FIG. 19.—Diagram showing all the quantities of matter which are needed for weighing any quantity between a milligram and 201 grams.

These fractions are, however, sometimes given in terms of the milligram, and they will then be 500, 200, 100, 100, 50, 20, 10, 10, 5, 2, 2, 1 milligrams. The quantities are of course the same, ·5 of a gram being the same quantity as 500 milligrams.

We shall find on consideration, that the set of masses enables us to cover much more ground in the direction of measurement than a scale would do, unless it were one of very inconvenient length. In fact, the above set contains 201,000 milligrams, easily arranged into any required quantity between those limits; whereas a metre scale contains only 1000 millimetres. That is, there are 201,000

different quantities of matter which can be made up from the box of weights; while the metre scale will only yield 1000 different lengths, unless we adopt some means of subdividing each millimetre, or of repeating the scale as in the operation of "laying off" the scale against another length.

EXERCISES IN USE OF METRIC MEASURE: MASS.

N.B.—*An account of each experiment must be neatly written out.*

1. Find out if the set of *weights* which are provided are correctly adjusted with regard to the gram, that is, find out if the mass called 5 grams is really 5 times the mass of the 1 gram, and so on for all the multiples of the gram.

It will first be necessary to find out if the 2 gram mass is twice the 1 gram, and this must be done by first counterpoising the gram with shot, etc., then substituting for the gram enough shot to maintain the counterpoise. This latter mass in the shape of shot, together with the gram, will be equal to twice the gram itself, and it will then be necessary to find out if the 2 gram mass can *take the place of* this combination without disturbing the counterpoise.

The stages of the operation may be represented as follows :—

(1) By experiment with a counterpoise, m. shot = 1 gram, since they can be substituted for each other on the same pan of the balance.

(2) Therefore m. shot + 1 gram = 2 grams.

(3) By experiment n. shot = m. shot + 1 gram.

(4) Therefore n. shot = 2 grams.

Now if the 2 gram mass from the box can be substituted for the shot we have called n. shot without disturbing the equipoise, then the 2 gram mass is equal to twice the gram, or, more correctly, to twice the mass called 1 gram by the maker of the set of masses.

Precisely the same operations are performed in measuring each of the remaining masses.

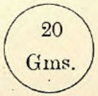
It is plain that if the 1 gram mass with which we started were known to be accurate, that is, known to be an exact copy of the standard mass, then we should have tested the whole set by the standard.

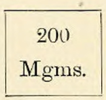
But it must not be forgotten that we are *dependent on the sensitiveness of the balance* for the value of the investigation. It is a question how slight an *inequality* the balance will detect. Probably the masses supplied may appear correctly graduated as far as we can judge from the balance used, in which case no fault can be found with them, *for they are as equal as they need be.* It would not follow that they would serve for use with a more accurate balance, for inequalities previously unobserved might then be discovered.

2. Construct a decigram and a milligram (if the balance permits) from aluminium foil, adjusting with the aid of a pair of scissors. The process of substitution in weighing, which has been previously used, is needed. The whole set of fractions of the gram may be made and stamped with their value for future use.

3. Write down the number of milligrams in 103·101 grams. What fraction of a gram is the sum of 7 centigrams and 11 milligrams? What number of milligrams added to 5 centigrams would make a gram?

4. Make a drawing of the masses used to counterpoise a body the mass of which is 28·47 grams. Draw circles for grams and squares for milligrams, for example:





The drawing must give a picture of all the "weights" you would use. (Do not forget that if you use the word *weight* you use it for the present with the same meaning as mass.)

5. Draw similarly 17·08, 7·77, 11·18, ·39, and ·06 gram.

6. Place out on the table the groups of masses needed to make up the following quantities:

1·7, 2·65, 3·01, 7·91, 9·99, 9·999, and 100·001 gram.

7. Assuming that the balance has been adjusted, and that the standard masses are correct, find out the mass required to turn the balance to the extent of one division of the scale, when the

pans are empty, and also when they contain equal loads. Vary the loads from 0 to 250 grams. Give the results in a table, which should be neatly written. For example:

Mass of Load on each Pan, in grams.	*Mass required to turn the balance.*	*Mass of Load on each Pan, in grams.*	*Mass required to turn the balance.*
Nothing	...	70	...
10	...	80	...
20	...	90	...
30	...	100	...
40	...	150	...
50	...	200	...
60	...	250	...

8. Two bodies of nearly equal mass are given. Find out the difference between them. Measure each separately for one method, and compare the result with that obtained by measuring the two together, and subtracting from that value the mass of one body alone.

9. Let a line 1 cm. long represent 1 gram, then draw lines equal to 15·7, 11·05, ·95, 7·09, and ·0999 gram respectively. Measure off on a straight line lengths equal to them, and ascertain the whole length of the line. What mass will this length correspond to? Test your result by adding up the masses.

10. Find out the mass, *in grams*, of $\frac{1}{2}$ lb. Weigh carefully, by the method of substitution. Assuming $\frac{1}{2}$ lb. to be equal to 227 grams, express the difference, between this and the mass you have found, as the fraction of an ounce. In other words, by what portion of an ounce does the supposed $\frac{1}{2}$ lb. differ from a true mass of $\frac{1}{2}$ lb.?

You will find out by (1) calculating the number of grams in an ounce, knowing both the number of grams and of ounces in $\frac{1}{2}$ lb., and (2) dividing the difference observed by the number found in (1).

11. Three bodies weighing respectively 5, 3, and 8 grams are given to you. By making use of these prepare another mass equal to 4 grams. You are not allowed to use weights.

12. Find out the mass which must be added to the counterpoise at the end of the graduated arc on the weighing instrument (Fig. 13), in order that the gram marks may be read as ounces (or the ounce marks as pounds).

PRECAUTIONS TO BE OBSERVED IN MEASURING LENGTH AND MASS.

LENGTH.

44. The essential of accuracy in measuring length is **to be certain when two lengths are equal.** In most cases we depend for this upon our *eyesight.* (In some cases, however, in the use of callipers and gauges for instance, *touch* may be adequate.) By means of the eyesight we have to determine when two points or marks coincide in position. No matter how difficult the operation, we cannot escape the direct appeal to our sense, and this may have to be made not once alone but frequently. It is therefore important to guard against **any causes which may render the coincidence of marks imaginary instead of real.**

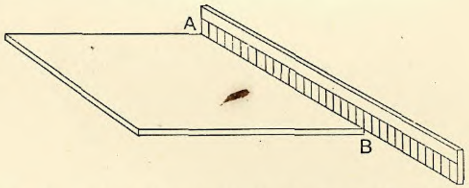


FIG. 20.—The distance between the two marks A and B is best measured by turning the scale on its edge.

The marks drawn upon scales have of necessity a definite *thickness*, and in consequence it is easy to make a considerable error in "taking off" distances from a scale, especially when the distance is small. For example, the thickness of a division line is not serious in a metre, but it is in the measurement of a millimetre. The centre or, better, the extreme edge of each line should be regarded always.

And on all occasions the marks to be compared should be made to *touch* if possible. For example, a scale should always be turned on that edge which will bring its division lines into actual contact with those to be measured (Fig. 20).

45. Again, it may happen that two marks cannot be made to touch. The eye may then make mistakes by reading sideways, instead of from the exact front. The danger is more frequent in the indirect observations which will come later, than in those direct and simple ones which we are now performing (Fig. 21). The change of a scale itself, due to curvature, is sometimes noticeable. Sometimes, too, in indirect measurement, the distances *supposed to be equivalent to the one under investigation* are far from being so.

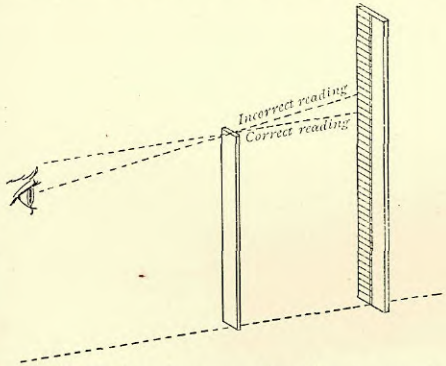


FIG. 21.—An illustration of an error of reading on account of the position of the observer's eye.

As an example of this, to find the distance between two points on a curved surface to which, from their position, a scale cannot be directly applied. Here it is necessary to make use of two parallel straight-edges, carrying on, as it were, the points to an accessible position. That the *transferred points* may retain their relative position, the lines along which they may be supposed to travel must be *parallel*. This demands care and safeguards.

The observation of the position of liquid surfaces in graduated tubes is often made erroneously. The height of a liquid in a burette, for example, should always be read by placing the eye on the same level, and reading, on each occasion, the mark which coincides with the *centre* of the curve made by the surface of the liquid (Fig. 22).

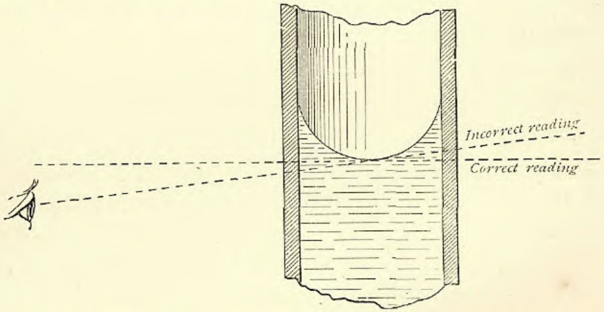


FIG. 22.—The two lines drawn show that the reading of the height of a liquid in a burette depends upon the position of the eye.

46. The importance of taking a number of readings of all measurements and then estimating the average or mean value cannot be exaggerated. This is done by adding them all together and dividing by the number of observations. The results should be written out as follows:

1st measurement	- -	= 8·93 cm.
2nd ,,	- -	= 8·95 cm.
3rd ,,	- -	= 8·94 cm.
4th ,,	- -	= 8·94 cm.
		4) 35·76 cm.
	Mean	= 8·94 cm.

MASS.

47. In using the balance the object to be measured should always be placed in the *left hand pan*, and the weights in the right hand pan, which is more conveniently reached. It is also important to use the same pan for the same purpose on all occasions: the results are then more accurate.

The masses in the two pans are more likely to be equal when the pointer swings an equal distance on each side of the scale, than when it is at rest at the centre. Dust or bad fitting may cause a balance to come to a position of rest where it would not otherwise be.

The weights *must be used in the order in which they are arranged in the box*, and must be picked up always by the tweezers. Bodies should not be placed in the pan except when the beam is supported. Carelessness in the use of weights is a serious misdemeanour in laboratories.

When weights are not in use on the pan *they must be in their right place in the box.*

NOTE.—The actual manipulation of a balance is most readily learnt in the laboratory by means of a practical demonstration. And it is advisable for the class to give some time to investigating the construction of a balance by taking it to pieces carefully.

GENERAL SUMMARY.

48. Most of the measurements and exercises set above are simple enough in themselves and do not require any but the simplest means of measurement; but, on the other hand, they would not be worth undertaking were it not that they form a valuable exercise in accuracy. Many mistakes may be made in the simplest observations, and the means of avoiding them will be learnt by practice alone.

The results shown up in the note-book must be neatly written, and **no number must appear without the name of the unit used.**

In most examples repeated readings should be taken, and the mean result calculated.

49. But besides the importance of learning to avoid errors of reading, there is an equally valuable lesson in finding out the possible limit of accuracy. Absolute accuracy is of course not attainable, for there will be *something over* in all measurements. But then ordinary measurements as a rule do not even approach to accuracy.

There is a *highest possible accuracy* dependent upon the circumstances of the measurement, and this accuracy should be attained as far as possible. For example, we can read very easily to the half of a millimetre, or less, by inspection, but this is useless if we have made an error of 2 millimetres elsewhere.

We may, for example, sometimes find that the terminal measurements of a scale are inaccurate to the extent of half a millimetre, especially with wooden rules. This may be remedied by measuring from the last accurate division in all cases.

Again, nothing is gained by using milligrams on a balance which will not turn with a decigram, but *some degree of accuracy* can be attained with such a balance, and this degree should be found out by experiment.

EXERCISE.—Write out as many examples as you can of the way in which errors may arise in making measurements, and state in each case how they may be avoided.

CHAPTER V.

MEASUREMENT OF SPACE—AREAS AND VOLUMES.

MEANING OF AREA.

50. It is not easy to understand completely what is meant by an *area* or *surface*, but many means of adding to our knowledge will suggest themselves in the process of measuring it. It will frequently happen that measurements need to be made without a thorough understanding of the quantity measured. If it were not the rule to carry on measurement by means of a quantity of the same kind as that being measured, this would not be possible; but since it is so, we can begin to measure surface by means of a unit surface, just as we measured mass by means of the unit mass, without knowing very much of the real nature of matter. We can begin to measure surface then, without having more than a rough practical knowledge of it. The total surface of a body is generally called its *area*, but the terms surface and area are often used one for the other.

Now so much may be stated about surface, that it is the place of separation between a body and the space in which it exists. Consider for a moment a sheet of paper. We have before us a certain quantity of matter which could be measured by the balance. It is separated from the air by six *boundaries* or *surfaces*, two of them being large and four being very small in comparison. These surfaces together make the piece of paper a separate and individual portion of matter. There is a total boundary, which, speaking for practical purposes, may be said to divide and set aside the air from the paper.

So too with any solid object, we can say that it is bounded by a surface of separation from the rest of space. **The surface does not really belong any more to the paper or other object than it does to the air or space.** It is neither paper nor air, for it is not *measurable except as surface.* It was just the same in measuring lengths, the *nature* of the body on which or through which the lengths were measured had no connection with them. It cannot, in fact, be treated as having *any existence* apart from the body which presents it, although an independent existence of surface can be *imagined*, just as an independent existence of length can be *imagined*. It takes up no room, nor can we rightly consider that it forms part of the object upon which we are going to measure it, any more than it does of the space around it; nor must we look upon it as a *very thin* covering, for it has **no thickness whatever.** We pass without any interruption or break from the object to the air; there is no neutral region.

51. Perhaps it may be stated here that a line is that which separates two surfaces from one another, or two portions of the same surface. Two surfaces may meet and make a natural line, the *edge* of the paper, for

example. Or we may draw a line on one of the large surfaces of the paper, which may divide that surface into two portions. We can *imagine* such a line to divide the surface and yet not take away from its quantity. And if we cannot imagine this, we can carry our thoughts to the extreme edge of our line, and say that a division of the surface occurs along that line. Just as **a line does not take up any surface**, or, in other words, does not take away from or in any way diminish the surface on which it exists; so **a surface does not take up any space,** nor does it take away from or diminish the space in which it exists.

Here again we must distinguish between an imaginary line and a mark made on paper to stand for a line; such a mark does take up surface. Although our ordinary measurements are not so accurate that we need take into account so small a length as the thickness of a fine line, yet a large number of the finest lines drawn side by side, so as to touch one another, would take up a perceptible quantity of surface.

EXERCISE.

Write out a description of what you consider a surface to be. State how a surface may be measured.

When a given surface is denoted, say the surface of a book, may we also regard this as a surface to the air or space around the book?

Does there exist any connection between the size of a surface and the kind of matter presenting that surface?

In common language we speak of rough, smooth, and other kinds of surfaces. Show that we are not at present concerned with such matters, but only with the quantity of surface, while we neglect those irregularities of surface which may be easily made very apparent by a lens.

What do you consider a line to be? Will a line which you draw on paper be a real line according to your description? Look at your line under a microscope, and make a drawing of its appearance.

MEANING OF VOLUME AND SHAPE.

52. The measuring of **volume** will be easily understood by those who have any clear ideas about surface; for they must always remember that **volume is the quantity of space enclosed by a real or imaginary surface or surfaces.** When we speak of the volume of a body, we refer to the space it occupies, to the room it takes up, to its bulk or size. All these phrases mean the same thing. Very often it is necessary to speak of volume quite apart from any object filling that volume; just as we can consider a length without thinking of the body upon which it has been measured, and as we can consider surface without attending to the body which presents it. We can add together or subtract volumes as we can any other quantities. If we could not *measure* volumes we should be unable to add them together or subtract one from another. It need not be stated that it is impossible to add areas to volumes, and we may here repeat the following statements:

1. A line has no surface, it possesses length alone, and makes a division upon the surface which presents it. A line may be drawn so as to enclose a surface.

2. A surface has no volume, it has length and breadth alone, and acts as a division of space. A surface may enclose a space in the same way as some lines may be regarded as enclosing surfaces.

3. A volume has length, breadth, and thickness; it occupies space, and is a portion of space.

53. Care must be taken not to confuse volume with **shape.** No one would be likely to think the **size** of a body the same thing as its **shape,** for two bodies may have the same shape and differ very much in size. If it be remembered that volume corresponds with size, there will be no risk in this respect. It is plain that a given body might change its shape without altering its volume; in other words, the surface may change and yet enclose the same quantity of space. Most solid bodies can be bent, twisted, or broken into pieces without undergoing a change of volume (Fig. 23).

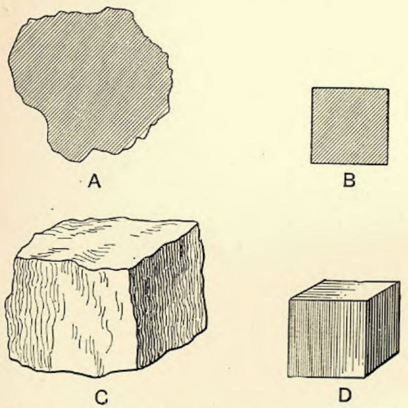


FIG. 23.—*A* is an area *equal to* three times the standard, or 3 square centimetres (*B*). *C* is a volume equal to four times the standard, or 4 times the cubic centimetre (*D*).

It will also appear that two bodies of different shape may have the same volume. This may be impressed on the mind by very simple means. We might take a certain number of bricks, and with them build up several bodies of very different shapes, yet the volume of all would be the same. That is, the quantity of space filled would be the same in each case, although the shape of the surface enclosing that space would vary. The following table will show the terms which need to be carefully distinguished :

AREA is quantity of surface.

VOLUME is quantity of space.

SHAPE is something which is independent of area and volume ; it cannot be measured ; it is not a quantity. (Shape may be of surface or of space ; that is, there is a difference in the shape of surfaces just as much as in the shape of volumes or spaces.)

It may be pointed out that **shape is not a quantity,** for it cannot be measured. **We cannot take a given shape and say this shape is so many times the standard shape.** *Shape can only be described.*

EXERCISE.

What do you understand by the term "Volume"?

By what reasoning would you consider volume to be a quantity, and shape not to be a quantity?

Write out a table showing all that has been stated about lines, surfaces, volumes, and shapes.

Give instances of change of shape taking place without change of volume, and also of change of volume occurring without any change of shape.

THE MEANING OF A PLANE SURFACE AND A STRAIGHT LINE.

54. There is one kind of surface called a plane urface, which must be discussed before we can proceed to make measurements of surface. It is not difficult to understand what is meant by a **plane surface,** or a **plane,** as it is often called, if we begin with a description of how such a surface is practically produced.

An account of the manner of making a plane surface serves the purpose of describing what is meant by the word plane.

Now the method used for obtaining a plane surface on a body is to alter its shape by scraping and rubbing, until it fits at all points with a known plane surface. To test whether it does touch at all points when placed on a standard plane, we may lightly cover the surface of the plane with a very fine powder, such as yellow ochre, mixed with oil. The other surface should now be placed on the plane, and on lifting it up again the manner in which the powder adheres will indicate whether it is truly plane or not.

55. But in order to obtain a plane surface when there is not one at hand for comparison, what must be done? We must take *three surfaces* and grind them together, until any two of them will fit over one another at all points. That is, each one must be able *to touch completely all over as much of the surface of each of the others as is equal to its own.* If there be a difference in size of surface, they must be tested by sliding over one another, and there must not be *any hollow spaces enclosed.* It is advisable to inspect a variety of surfaces, so as to understand the meaning of this statement, and it is well to make an effort to prepare from some fairly soft material a true plane surface.

Finally, we may say that **each side of a plane is of exactly the same shape.** It is worth while trying to understand this statement, for much depends on its truth. It may be represented in a fashion as follows: The first of the lines below (Fig. 24) divides the surface around it, which is supposed to be plane, into the same shapes. The second does not. It is clear that the surfaces to the right hand and to the left are alike in the first and unlike in the second diagram.

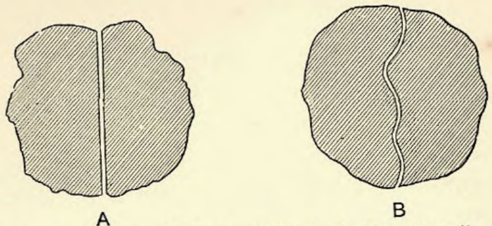


FIG. 24.—The area *A* is divided similarly by the line across it, while the area *B* is not.

They fit into one another, it is true, in each case, but in the second case it is only necessary to imagine them pulled apart to see they are unlike. The two terms, concave and convex, describe two surfaces which may fit one another and yet be dissimilar.

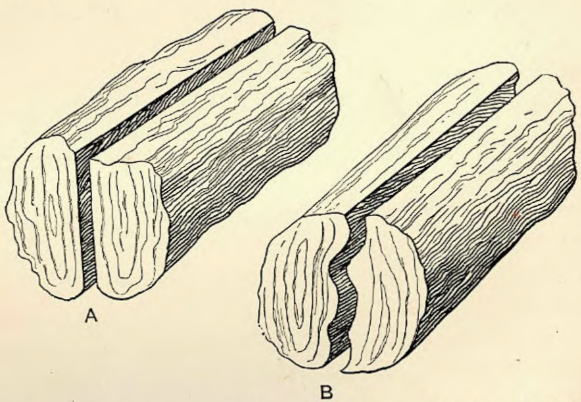


FIG. 25.—The space marked out by the figure *A* has been similarly divided by the plane cutting it, while the parts of the space *B* are dissimilar.

Now if we consider these lines to stand for *surfaces running through the paper at right angles*, and the

surfaces around them to represent spaces, as shown in Fig. 25, we shall understand that the surface is the same on each side of the plane.

56. At the same time we shall have learnt what is a straight line. It is a *line which lies on a plane surface containing it*, in such a way that the edges of the plane, if it were cut by the line, would be alike in every respect. It will also be perceived that when two planes cut one another, the edge or direction of meeting is a straight line (Fig. 26). (Practical

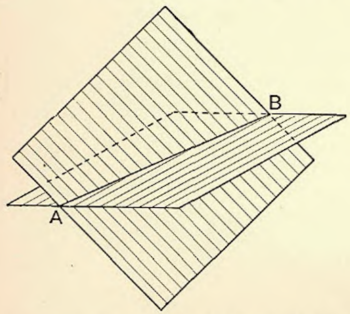


FIG. 26.—Two planes cutting one another, and forming a straight line *AB*.

illustrations of this with paper or cardboard should be given.) It follows from what has already been said, that where one plane cuts another the line formed must be straight; because both sides of any plane are everywhere alike, and therefore all sides of the line which is made where the two planes cut must be alike.

The two edges of a plane cut by a straight line would fit at every point, and permit of sliding without ceasing to make complete contact at every point which is common to the two edges. **A straight**

line then divides a plane so that each edge fits at all points, in much the same fashion as two planes fit one another.

The practical test for a straight-edge will show what is meant by a straight line. The edge is laid on paper and a line drawn to correspond with it. The edge is then reversed and placed so as to face the line. If the edge and the line now coincide, it is a true edge. Or again, a line is drawn by hand as nearly straight as possible. It is then traced through thin paper. The traced line is reversed and brought to face the first line, and made to approach it closely. It will then be very easy to see if the line be straight, for if not straight it will be able, with the aid of its counterpart, to enclose a space.

57. We may now proceed to state, that when one straight line standing on, or cutting another, divides the plane surface containing them into the same shapes, then these lines make **right angles** with one another. The figure below illustrates this.

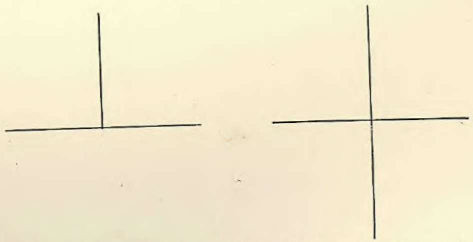


FIG. 27.—Two right angles and four right angles formed by two straight lines.

In the first case (Fig. 27) we have two right angles formed, and in the second one we have four

right angles. Paper should be cut with straight edges by scissors to impress this definition. In each case the right angles may be shown to fit one another, and to possess the same shape on both sides.

We may also arrive at some knowledge of the meaning of a right angle, by the practical method of drawing one upon paper by the use of compasses. A circle is described, and its diameter (the straight line passing through its centre and terminated by the circumference) is drawn. The circumference is now halved by the diameter, and it is only necessary to bisect the half circumference by the usual method, and the straight line drawn from the point of bisection to the centre of the circle makes two right angles with the diameter (*i.e.*, the line is *vertically drawn*). If produced there will be four right angles constructed, and we perceive that a *right angle subtends or stretches over the quadrant or quarter of a circle.*

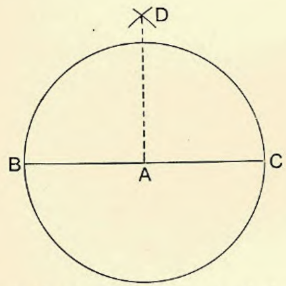


FIG. 28.—Method of drawing one line vertical to another.

METHOD.—At the point A describe a circle and draw a straight line BC passing through the centre A. Then with any distance longer than BA, describe a portion of a circle at D from B as centre, and likewise describe a portion of a circle of the same size at D from C as a centre. The straight line joining DA is the vertical required (Fig. 28).

The connection between angles and circles must be left to a later section, but we may here point out that an angle is formed by the **meeting or inclination of two lines**; and its value is measured by the distance one line is turned away from the other.

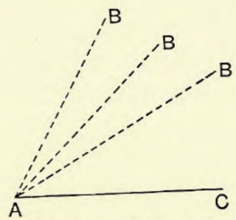


FIG. 29.—Diagram showing the meaning of magnitude as applied to an angle.

For example, as the line AB is turned round on the point A, and the point B moved further from C, so does the size or magnitude of the angle increase. **The length of the lines AB and AC have nothing to do with the size of the angle** (Fig. 29).

EXERCISE.

How may a plane surface be prepared? What is meant by saying that each side of a plane is of the same shape? Give some description of what you mean by saying two objects, whether solid bodies or surfaces, are the same shape.

In what ways does a straight line differ from a curved one? What is an angle? What is a right angle?

DESCRIPTION OF SOME REGULAR SURFACES AND SHAPES.

58. The great majority of surfaces and shapes to be met with in nature are very irregular. It is only when we come to Geometry and Mensuration that we find areas and shapes which are easily described and measured. The commonest regular surfaces are those enclosed by circles, ellipses, squares, triangles, rectangles, and parallelograms. The most familiar shapes are the cube, oblong, sphere, cylinder, and cone.

The method adopted by Euclid in dealing with space is to define a point, a line, and a plane surface, and from these as starting-points various angles and figures are described and defined.

A **point** is described as "that which has position, but no magnitude."[1] A point is that which marks position and has no size or shape.

A **line** is "that which has length without breadth. It therefore marks length and direction." A **straight line** is "that which lies evenly between its extremities."

An axiom is expressed after this definition by Euclid, who says, "that two straight lines cannot enclose a space." This **axiom** or **self-evident truth** is really a part of the description of a straight line, and helps us to understand what it is, for we should not otherwise know what is meant by "lying evenly between its extremities."

A **surface** is "that which has length and breadth but no thickness, and the boundaries of surfaces are lines."

A **plane surface** is "one in which any two points being taken, the straight line between them lies wholly in that surface."

[1] Refer to the definitions of Book I. in Euclid, and read them through carefully, bearing in mind what has been previously stated in this book.

59. This description of a plane surface differs completely from that previously given, but it is none the less true. A straight line has been defined as the intersection of two planes, but Euclid first explains what is meant by a straight line, and then proceeds to describe a plane surface, by making use of the property of a straight line as previously assumed. In fact he *tests the plane by means of straight lines,* whereas we have so far derived our knowledge of a straight line from the description of a plane.

But although there may be different views as to the best definitions of straight lines and planes, there can be no doubt about the clearness with which the subsequent definitions are derived by Euclid from these preliminary definitions.

A **plane angle** is "the inclination of two straight lines to one another, which meet together but are not in the same straight line."

(Strictly speaking, we might regard the angle formed by two lines which *are in the same straight line* as having a magnitude equal to two right angles.)

It is interesting to read the definition of a right angle, which is based upon the fact that one straight line may stand upon another so as to make, together with this line, two equal angles. These are *right angles.*

The definitions of all the plane figures should be learnt from Euclid.

In the next place, all these surfaces should be *drawn* and then *cut out* in paper. It can then be shown:

60. 1. That a circular surface sweeps out a sphere if made to turn on its diameter.

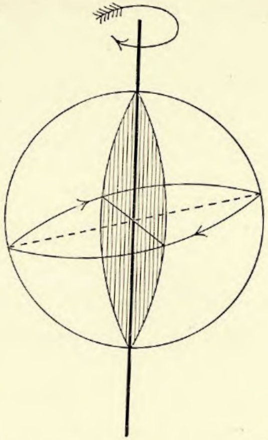


FIG. 30.—A sphere produced by a circular surface turning completely round about its diameter.

2. That a cubical space is traced out by moving a square in a direction perpendicular to its own plane through a linear distance equal to its own side. ($AB=CD$ in Fig. 31.)

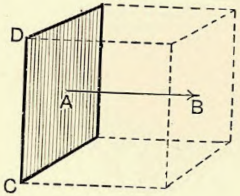


FIG. 31.—A cube traced out by a square surface moving through a distance AB equal to its side CD.

3. That an oblong is traced out whenever a rectangle or square is similarly moved, through *any* linear distance, perpendicularly to its own plane.

A practical illustration of a plane surface moving perpendicularly to itself is important. It may be noted that each point in the plane would move in a direction, which is at right angles to any straight line in the plane.

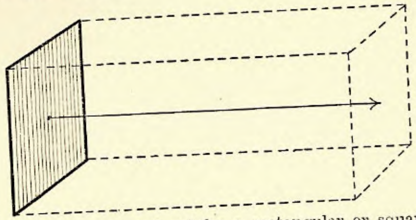


FIG. 32.—An oblong traced out by a rectangular or square surface moving through any given distance.

4. That a right cylinder may be traced out by the turning of a rectangle about any of its sides.

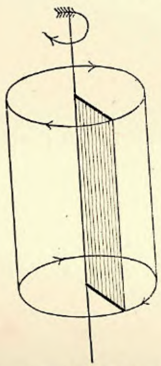


FIG. 33.—A right cylinder traced out by the rotation of a rectangular or square surface about one of its sides.

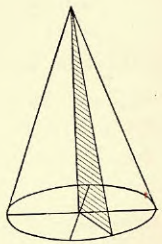


FIG. 34.—A right cone traced out by a right-angled triangle.

5. That a cone is formed when a right-angled triangle rotates about one of the sides containing the right angle.

All these operations should be imitated and the

shapes should be imagined, as completely and vividly as they can be.

The fact that a triangle is half the parallelogram which has any two sides of the triangle as its adjacent sides, is readily shown by two equal triangles in paper. Also, that the area of a parallelogram is the same as that of a rectangle on the same base and having the same altitude, may be shown by cutting out a parallelogram in paper, and then cutting off a right-angled triangle from one end, which will be seen to fit the other end.

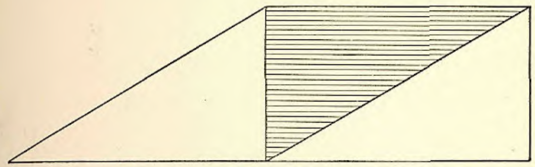


FIG. 35.—The relation of a rectangle to a parallelogram on the same base is seen, and also that of a rectangle to a triangle.

61. It now remains to define the units of area and volume. **A square, each side of which is 1 cm., forms the unit of area; and a cube, each surface of which is 1 sq. cm., forms the unit of volume.**

It is not necessary, perhaps, to state again that *the numerical values of area and volume are quite independent of shape*, and that when we say a surface has an area of 3 sq. cm. **we do not imply that it has a resemblance to 3 squares,** nor does the statement that a body has a volume of 6 c.c. give any information whatever about the **shape of the body.**

The units above mentioned are not the only ones in use in this country. A table gives the values of other units, section 79.

PRACTICAL ILLUSTRATIONS OF THE MEASUREMENT OF AREAS AND VOLUMES.

62. In order to obtain a clearer understanding of the unit of area and the use made of it in measurement, a square having each side 1 cm. in length should be carefully constructed on paper, and the area then cut out with the help of scissors. The advantage of a connection between the unit of length and the unit of area is clearly very great.

In the same manner the unit of volume is related to those of length and area, for the unit of volume bears the unit area on each of its *faces*, and the unit length on each of its *edges*.[1] The straight lines separating the six faces from one another are each 1 cm. long.

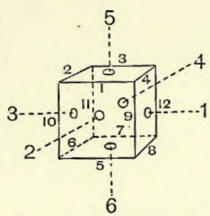


FIG. 36.—A cubic centimetre with 6 square centimetres of surface, and 12 edges each 1 centimetre long.

A square, each side of which is 4 cm. long, may now be constructed and divided up into squares of 1 sq. cm. The same process may be carried on with a square of 6 cm. in the side, and also in the case of a rectangle which has sides of 6 and 4 cm. The numbers of squares equal to the unit square may be *counted* in each of the figures, and it will be seen

[1] Wooden cubes should be handed to the class, cubes of 1 c.c. by preference.

that the *same result* may be obtained by the process of *multiplying* the value of one side of the square or rectangle by the value of the other side. For example :

cm. cm.				
$4 \times 4 = 16$	and this square contains		16	*square centimetres.*
$6 \times 6 = 36$	,,	,,	36	,,
$6 \times 4 = 24$	,,	,,	24	,,

The change from linear to square measure is brought about by multiplying two linear dimensions together. Length is multiplied by breadth, and the result is the numerical value of the area in question.

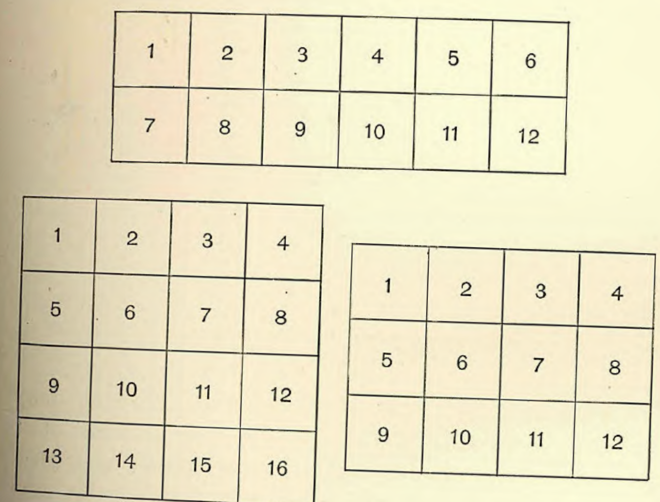


FIG. 37.—A square of which each side is 4 centimetres in length, containing 16 square centimetres, and two rectangles of equal area (each of 12 sq. cm.).

63. But we need to draw attention to the fact that this method of judging the areas, by multiplying length by breadth, can only apply directly to squares and rectangles. It is true that it may be applied to parallelograms, but in that case we must take care to multiply the length, not by the length of the other side, but by the *vertical height*, which is the real breadth.

The following figure (38) will illustrate this point, and especially so if it be copied out in your note-book, making the parallelogram a *whole* number of cms. in length and also in height. The lengths to multiply are AB and EC, for it is easily seen that the area $ABCD$ is equal to the area $CDEF$.

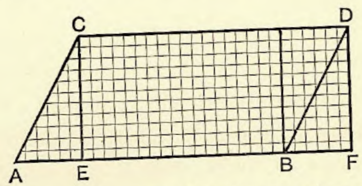


FIG. 38.—A method of proving that the area of a parallelogram is obtained correctly by multiplying the numerical value of its length AB by that of its height EC.

When we come to deal with less regular figures, and with figures having boundaries which are not straight lines (circles, ellipses, etc.), other methods must be used.

64. It is as well to make sure that the above method of multiplication serves when the dimensions contain fractions. This can be done by drawing a square with each side 5·5 centimetres. Then add together the whole squares and likewise the half squares. The sum will be the same as that obtained by multiplying 5·5 by 5·5.

The same proof may be adopted with other fractions, and also in the case of rectangles. Exercises should be taken with this proof in view.

EXERCISES.

1. The side of a square is 3·9 cm.; what is its area?
2. What is the area of a rectangle which is 6·2 by 5·5 cm.?
3. What is the area of a triangle which is 7 cm. in height and has a base of 6 cm.?
4. A parallelogram has sides of 6 and 4 cm.; find out by construction what its area will be.

Note.—Each of the above areas should be drawn of the right dimensions, and the units of area illustrated as well as calculated.

65. The methods of calculating volume come next under consideration. They resemble the calculations of areas, with this difference, that instead of multiplying **two dimensions** together, we need to multiply together **three dimensions,** namely, *length, breadth,* and *thickness,* in order to obtain the numerical value of **volume.**

In order to realize the relations between *linear, square,* and *voluminal* or *cubic* dimensions, small wooden cubes should be obtained.[1] These may be built up into cubes of various sizes.

It will be found that if the cube has a side of 2 cm. then it contains 8 c.c. (or $2 \times 2 \times 2$), whereas if the side equal 3 cm. there are 27 c.c. altogether (or $3 \times 3 \times 3$). The face of the latter cube will be found equal to 9 sq. cm.

Several illustrations of this kind should be thoroughly investigated by the class, and in doing so the meaning of the term **"the three dimensions of space"** may become clearer. Bodies constructed with three different linear dimensions afford still more valuable lessons, as

[1] These cubes should be obtained in large quantities from a merchant who supplies the common wooden toy bricks.

the distinction between the three dimensions is more apparent.[1]

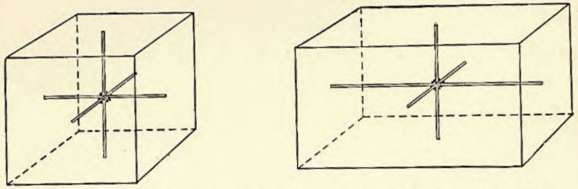


FIG. 39.—Diagram illustrating the *three* dimensions of a cube or oblong.

66. But in the case of volumes, just as much as in that of areas, the possibility of *calculating values* from the other known dimensions does not extend very far. There are a certain number of regular figures in which this can be done, as may be easily demonstrated by the model cubes, for example, cubes and oblongs. And there are a certain number which can be indirectly calculated, spheres and cylinders, for example.

But the ordinary bodies met with in nature, just as the ordinary areas, do not conform to the regularity of the figures we have been discussing, and other means of measurement have often to be adopted. Examples of these will be given in subsequent exercises.

FURTHER PRACTICAL ILLUSTRATIONS OF THE MEASUREMENT OF AREAS.

67. Before passing to these, much more may be learnt of the measurement of areas in a practical manner by means of paper and a pair of scissors.

[1] Models of the three dimensions of cubes and rectangles should be constructed by the class, by means of sticks and twine, as shown (Fig. 39).

Cut two pieces of paper to contain two right angles, by means of taking two pieces and cutting them together. Make several efforts until the equal angles, which you have obtained in this way, form a perfectly straight line when placed back to back. They are *not right angles unless they stand this test, for two right angles, lying adjacent, must form a straight line.*

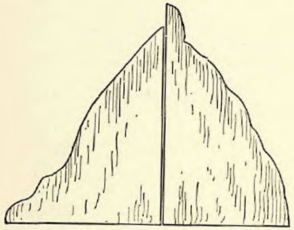


FIG. 40.—Two right angles of paper, lying adjacent and forming a straight line.

We may next proceed to cut four angles together, and test whether they are right angles by placing the four angles together. If they do not fit together *without leaving spaces* they cannot be right angles. Any two right angles together stand on a straight line, and *straight lines may be made to coincide in direction.* Hence no spaces can be left when four right angles are placed together.

68. The method of finding the area of a parallelogram by multiplying the length by the vertical height may be readily shown by cutting out in paper an accurately constructed parallelogram. Now draw from one corner such as C (Fig. 41) a line CE perpendicular to AB. Then cut off the portion ACE, and fit it on at the other end. We now have a rectangle $CEDF$ formed out of the parallelogram $ABCD$. The area of the rectangle is known to be obtained by multiplying the length by the breadth, and this is *the same as the area of the parallelogram.*

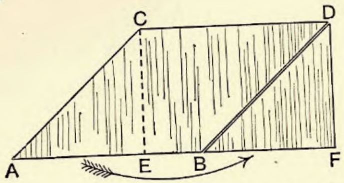


FIG. 41.—A portion of a parallelogram in paper, which has been cut from one side and fitted to the other side, changes the parallelogram into a rectangle. Note that any parallelogram can be converted into a rectangle by *one* cut, together with a change in position of the parts.

The manner of calculating the area of a triangle may be understood by cutting out two triangles together so as to be equal. One triangle is then reversed, inverted, and placed against the other so as to form a parallelogram (Fig. 42).

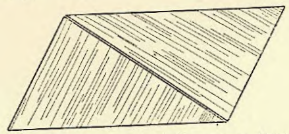


FIG. 42.—Two equal triangles, cut at the same time from a doubled sheet of paper and placed together, form a parallelogram.

Now we have learnt (1st) how to calculate the area of a parallelogram, and (2nd) we know that any triangle is able, when doubled, to form a parallelogram. Hence we can calculate the areas of triangles.

The proposition of Euclid (No. 35, Bk. I.), which affirms "parallelograms on the same base and between the same parallels to be equal," may be readily demonstrated in more than one way. The whole figure may be first cut out, then the two parallelograms are separately cut out so as to be equal to those in the first figure, and also the two triangles which may be seen to form part of the two parallelograms. The

proof as given by Euclid is easily followed by the aid of these figures in paper (see Fig. 43). *This proposition is one alone of a large number which may be quickly understood by means of figures cut out in paper.*

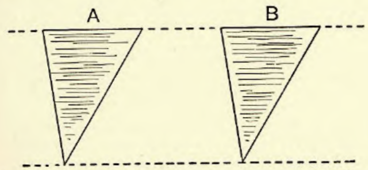


FIG. 43.—Parallelograms on the same base and between the same parallels are shown to be equal by figures cut out in paper. Two similar triangles *A* and *B* are cut out, and are moved through any distance between the same parallel lines (shown by dotted lines). At any relative position of the triangles, mark the paper underneath to correspond with the inner side of either triangle, and remove that triangle. A parallelogram is left partly formed of dotted lines, and partly of the remaining triangle.

69. It will be instructive to divide a circle into a large number of equal sectors, by measuring equal

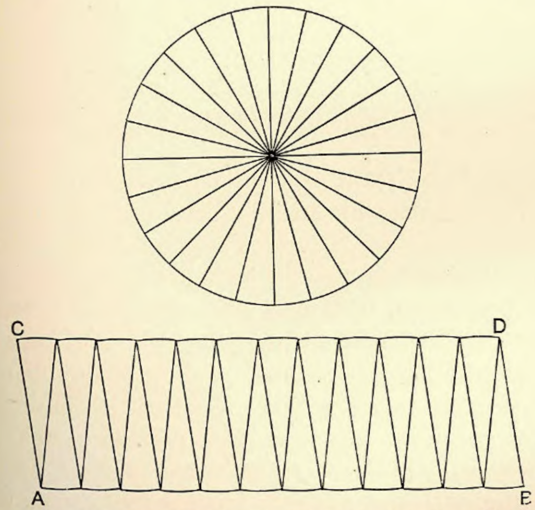


FIG. 44.—A circle divided into a *large number* of equal sectors, which, when placed side by side as in *ABCD*, form approximately a rectangle.

distances along the circumference, and cutting from these points along radii to the centre of the circle. These may then be placed together so as to form a surface of the shape shown in Fig. 44.

The area of this figure may be found approximately by *treating it as a rectangle.* The *smaller* the sectors the *more closely* will it approximate to a rectangle. The area, as measured in this way, will be found to be $3{\cdot}14 \times r \times r$, where $r =$ the length of the radius.

It is clear that the side $AB =$ one-half the circumference, and since the circumference is 3·14 times the diameter, then $AB = 3{\cdot}14$ times the radius or $3{\cdot}14 \times r$. Hence approximately $AB \times AC = 3{\cdot}14 \times r \times r$.

PRACTICAL ILLUSTRATIONS OF THE MEANING OF STRAIGHT LINES AND PLANES.

70. We may now extend still more widely our knowledge of straight lines and plane surfaces, by dealing practically with objects which are supposed to exhibit them.

A straight-edge, for example, is a rod or bar of wood or metal with an edge which presents a straight line. It is used for testing other straight surfaces, or to enable a straight line to be drawn.

We may rapidly test in a rough fashion whether the edge is straight, by drawing a line from it on paper. The straight-edge is then *reversed*, and a line drawn from it to face the first line. If the straight-edge be really straight, the lines may be seen to be parallel, and might be made to coincide; but if it be not straight, the two lines will probably take one of the appearances shown in Fig. 45 in an exaggerated form.

An ordinary metre scale or yard scale is very liable to be wanting in accuracy at the edges; and frequently the want of straightness is easily observable when you look along the edge with one eye.

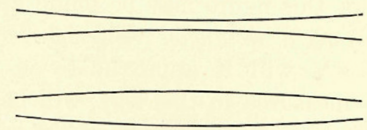


FIG. 45.—Appearances presented by lines drawn from an object which has not a straight edge.

The method just carried out will only detect a fault in the straightness of the edge when used in one direction. The edge may be straight in this direction, and crooked in a direction at right angles, as Fig. 46 shows. The edge looked at from above may be straight, but not when it is looked at sideways. The straightness from this side may also be tested.

A similar test, by means of lines drawn from that edge in reversed positions, will afford an indication of its straightness. *This should be practically carried out.*

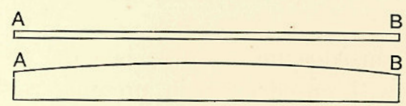


FIG. 46.—An edge which is straight when regarded from one side, but curved when regarded from a side at right angles to the former.

71. The eyesight is often a sufficient test, not only of the straightness of edges, but also of the flatness of surfaces, and whether two surfaces are at right angles to one another. But the test becomes more certain if we are able to **magnify the errors** in any way.

In the case of a bar AB (Fig. 47), if it is uncertain whether the whole of the upper surface is in the same plane, we may ascertain it with fair certainty by placing two laths, C and D, each having quite straight and parallel edges, on the bar as shown. (The steel scales used in measurement will serve this purpose

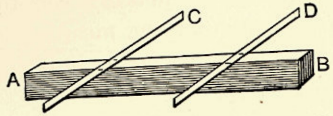


FIG. 47.—A method of testing whether the surface of a bar is a plane surface.

very well when propped up.) By placing the eyes on a level with the edge of C, and looking along the length of AB towards D, it will be easy to see whether the edges of C and D may be made to coincide by altering the level of the eyes. Any variation of the surface from a true plane will be rendered much more apparent by these means.

A similar method may be followed in testing if two surfaces are at right angles to one another. It is only necessary to extend those surfaces, by using straight-edges, for any deviation from a right angle to become

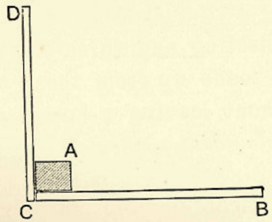


FIG. 48.—A method of testing whether two surfaces are at right angles to one another.

apparent. The object A may be placed on a known straight-edge or plane BC, and another straight-edge DC brought in contact with the other face. It will be easy to see if the line BCD forms a right angle (Fig. 48).

72. But these experiments introduce us to more systematic operations. The straightness of an edge or the trueness of a plane surface may be tested with still greater accuracy; and in making the tests we shall learn in a practical manner the meaning of a straight line and a plane surface, so far as it is connected with the material work of the world.

There are certain descriptions of lines and surfaces with which you are concerned in learning Euclid and Geometry. But the ways in which you will regard such matters in those studies need not interfere with your practical investigations; **though Geometry is based upon practical experience.**

But it seems more natural that you should learn in the laboratory ideas about lines and surfaces, which not only are true in themselves, but are also derived from those methods of preparing plane surfaces and straight edges which are adopted in every workshop. And it may be added that these methods are adopted from necessity.

There is no other method of testing straight edges (except by using a standard, that is, *a known* straight-edge), **but by selecting any three of them, and finding out if any pair, made up from those three, be able to fit together without leaving a space.** *The same statement may be made about planes.*

73. This practical manner of regarding straight lines and surfaces will be illustrated in greater detail, if we attempt to follow, as closely as is possible in a *laboratory*, the methods of obtaining them in the *workshop*.

Three rods of wood, A, B, and C, are obtained. They are roughly straight and square, and about 8 inches in length, not more than a quarter of an inch in thickness, and of sufficient breadth to be fairly rigid. It will be sufficient to render one of the narrower faces of each rod perfectly plane. To do this the three faces are indicated by a mark. *A* is placed on *C*, and inequalities of surface will be seen on holding up before the light. These must be gradually removed by using emery paper wrapped over a stick. When the edges of *A* and *C* have been made to fit after repeated trials, the edge of *C* is placed in contact with *B*, and the same process is carried on till *C* likewise fits *B*.

We have now reached the stage that both *A* and *B* fit *C*, but we cannot yet say that they are all straight. They may fit in the manner represented in an exaggerated form in Fig. 49.

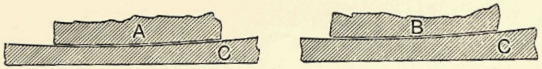


FIG. 49.—Two surfaces *A* and *B* both fitting a third, *C*, although they are not plane surfaces.

Then comes the next stage of the process. *A* must be made to fit with *B*. So far as we know, an appearance shown in Fig. 50 might follow. Any irregularity must be corrected in each, so that they fit one another. *But how do we know now that B and A may not be fitting in the same irregular manner as shown in Fig.* 49? We can only ascertain this by constant reference to *C* and *as B and A are each being rubbed down to fit one another, C must also be altered to fit them.*

Thus we have a succession of cautious trials and constant reference of *A, B, and C* to one another. Care and patience must be exercised over this important experiment until **all three fit one another.**

Then, and not till then, will they be **straight-edges** presenting **straight lines.**

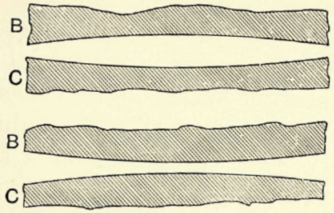


FIG. 50.—The effect of placing two surfaces together when they are not plane; yet each of them fits a third surface, as shown in Fig. 49.

74. On reference to the description of a straight line in section 56, we find that we have practically obtained that which was there said to be the essential property of a straight line, namely, **that it should divide the surface containing it into parts possessing edges, which fit and are alike at all points.**

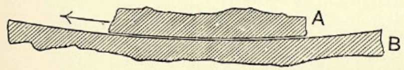


FIG. 51.—Two surfaces which are not plane sliding over one another on account of possessing the same curvature.

Now it is quite true that two edges may fit, as shown in Fig. 24 B, and yet it is only necessary to *slide*[1] the one over the other to show that they do not fit at all positions.[2]

[1] By sliding is meant the motion of one body over another in any direction, without the area in contact diminishing.

[2] The method of testing planeness by sliding, although fairly safe in practice, is subject to error. When two bodies, *A* and *B* (Fig. 51), have a surface of the same curvature, they will slide over one another without showing vacant spaces. But these surfaces are not planes.

But the test is *much more complete* when we use *three separate edges*, for if they all fit one another, we may say that a surface represented by the faces of two bodies (as shown in any of the three cases in Fig. 52) has been divided by the line *DE* in such a manner that the two edges of the surface fit and are alike in every way.

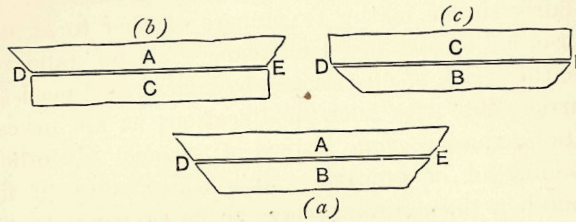


FIG. 52.—Diagram illustrating three plane surfaces *A*, *B*, and *C*, fitting one another.

By what reasoning may we assume this to be true? The steps in the argument are as follows:—*B* is shown to fit *A*, and *C* is shown to fit *A*. Likewise *B* is shown to fit *C*.

We have shown, therefore, that the place of *B* in (*a*) can be taken by *C*, and we obtain (*b*). **We learn that the same side of the line *DE* is fitted equally well by *B* and *C*.**

But if we now place *B* and *C* together as in (*c*), we come to the decisive test. **If *B* and *C* now fit, we have shown that both sides of the line *DE* are alike, for *C* will fit equally well on either side of the line *DE*.**

C has been placed below the line in (*b*) and above it in (*c*), while the line has been maintained in the first case by *A* and in the second case by *B*, and that *this is the same line* is proved by *A* and *B* fitting, as shown in (*a*).

75. The *same reasoning* and a *similar practical operation* may be carried on in *preparing plane surfaces.* In the workshop this is a long and laborious process in the first instance, when there is no standard plane for reference, and especially so, as they need to be made of metal.

When an accurate plane is already in existence, it is a fairly simple matter to prepare another to fit it; but in order to obtain a true plane when no standard exists, the series of operations above described need to be carried out, with such modifications as are necessary to produce a *plane* instead of an *edge.* In other words, instead of preparing three thin edges to fit one another, three surfaces have to be prepared to fit at all points.

The plates are prepared as accurately as possible in the first place by the use of a planing machine or file. Their fitting is then tested by covering the surface of one with a mixture of fine ochre and oil, and carefully placing another upon it face downwards. On lifting it up again blank spaces will be perceived in the covering of paint on the face of the second. A scraping tool must now be used to lower the portions of the surfaces which are covered with paint.

The plate is again tested, finally made to fit, and the third plate is made, by means of the same operations, to fit each of the others. Repeated trials and continuous work are needed to produce this result.

76. In order to imitate in the laboratory the preparation of a plane surface, **remembering that to produce one such surface, it is necessary to produce two others at the same time,** some soft material should be used. Cubes of *chalk*[1] or of *dried modelling clay* serve the purpose well, and three plane surfaces may readily be prepared by rubbing two in succession against the third.

The practical test that the surfaces of such a material are in contact at all points is not easy to make, but by carefully sliding one over the other, and at the same time watching the edge of the one and the surface of the other, any want of contact may be seen.

In the operations above described, it must not be forgotten that although the methods themselves are perfect, yet the results are only approximately perfect in the most carefully prepared plates. In

[1] The pieces of chalk which are used at billiards give an excellent illustration of the process, but modelling clay, which has been roughly made into cubes and then dried in an oven, serves the purpose very well. One edge of each face which is to be made plane must be made approximately straight by a

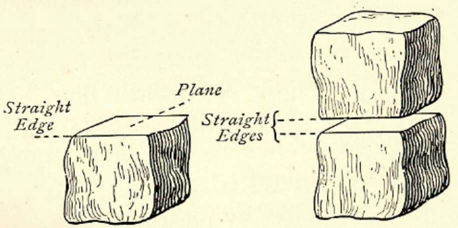


FIG. 53.—An illustration of three plane surfaces prepared by rubbing each in turn against the other two.

preliminary grinding. For without this edge being straight, it is not easy to perceive whether the surfaces fit when slid over one another.

In spite of the inconvenience of the dust produced during this experiment, *the lesson conveyed is so valuable that it should be carefully carried out without fail by the whole class.*

the models prepared in class, the results are far from perfect on account of the material which is used. Nevertheless, by similar methods, the best planes are prepared, and when prepared they exhibit very characteristic properties.

When one such plate is placed on another it is some time before they actually touch, as a layer of air becomes imprisoned between them, and when this has been gradually excluded they stick together so much that it is difficult to separate them.

EXAMPLE OF MEASUREMENT OF VOLUME. TO COMPARE THE CAPACITIES OF TWO VESSELS.

77. In comparing the *capacities* of two vessels we again give an illustration of that important principle of measurement, that in order to *compare* we must in the first place *measure*. At the same time a standard quantity must be selected before we can proceed to measure.

The same order is maintained in measuring volumes as in measuring other quantities. Something to measure by is selected, and by its use a numerical value is obtained. One quantity may be *compared* with another, by means of the numbers obtained in using the same standard.

Two beakers are provided, a large one and a small one, and it is required to compare their capacities. Now the first *standard* that will naturally suggest itself is the capacity of the smaller beaker, and we can find the capacity of the larger in terms of the smaller, by filling the smaller with water and emptying it into the larger, doing this over and over again until the larger beaker is full. But it is very probable that when the larger beaker has been filled some water will remain in the smaller beaker. This shows that the capacity of the smaller beaker is *too large a standard to use for the comparison.*

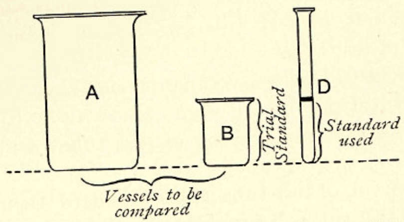


FIG. 54.—Two vessels, *A* and *B*, of which the capacities are to be compared. *B* is the standard first tried. The volume of the test tube up to the mark *D* is the standard which successfully measures both vessels.

The next best standard to use will be the volume of water remaining in the small beaker, because we shall then be making a practical use of the arithmetical rule for the Greatest Common Measure, just as we did in comparing two lengths (Fig. 3).

This standard can be fixed by pouring the water left over into a small test tube, and marking the height of the liquid in the tube with a piece of gummed paper or a small elastic band.

We have now to compare the capacity of *the smaller beaker* with that of *the test tube up to the mark.* We do so by filling it up to the mark with water and emptying the water into the small beaker, and repeating the process until the small beaker is full. If any water now remains over in the test tube, we can estimate fairly accurately what fraction of the volume of the marked portion of the tube it occupies, or, if we like, measure the actual

fraction by a process identical with that already described. Enter thus:

Large beaker filled nearly 4 times from smaller.

Remainder in small beaker (= volume of marked test tube) is poured into small beaker 5 times in succession. The remainder left in test tube was $\frac{1}{3}$ of volume of marked portion of test tube.

Volume of large beaker = volume of small $\times 3 +$ (vol. of small $-$ vol. of test tube).

Volume of small beaker = vol. of test tube $\times 4 +$ (vol. of test tube $- \frac{1}{3}$ vol. of test tube).

Taking $\frac{1}{3}$ of vol. of test tube as the standard, then we have

Volume of test tube = 3 times the standard.

Volume of small beaker = $12 + (3 - 1) = 14$ times the standard.

Volume of large beaker = $14 \times 3 + (14 - 3) = 53$ times the standard.

This result may also be expressed as follows:

$$\frac{\text{Volume of large beaker}}{\text{Volume of small beaker}} = \frac{53}{14} = 3{\cdot}8.$$

(Compare this operation with that of comparing two lengths shown in Fig. 3, and note that we find the G.C.M. of two volumes.)

78. In this experiment we have found the *relative capacities of the two vessels,* that is, we have found the volume which each contains in terms of the same standard. This standard is contained 53 times in the large vessel and 14 times in the smaller. These numbers indicate the relation existing between the volumes. Lines of corresponding length would equally well indicate this relation.

If the standard used were the unit agreed upon for general measurement, namely *a cubic centimetre,* then the numbers obtained would not only give the *relative value* as between the two objects measured, but we should know the relation existing between these and all other quantities which have been measured in terms of a cubic centimetre. It is clear that our knowledge becomes much wider *and more useful* by using the same unit on all occasions.

TABLES RELATING TO AREAS AND VOLUMES.

79. *English Areas.*

1 square mile contains 640 acres.

1 acre contains 10 square chains. (1 chain contains 100 links or 66 feet.)

1 acre contains 4840 square yards.

1 square yard contains 9 square feet.

1 square foot contains 144 square inches.

Metric Areas.

1 square metre contains 100 square decimetres.

1 square decimetre contains 100 square centimetres.

1 square centimetre contains 100 square millimetres.

English Volumes.

1 gallon contains 4 quarts.

1 quart contains 2 pints.

1 pint contains 34·659 cubic inches.

Metric Volumes.

1 litre is equal to 1 cubic decimetre.

1 cubic decimetre contains 1000 cubic centimetres.

1 cubic centimetre contains 1000 cubic millimetres.

COMPARATIVE TABLE OF LENGTHS, AREAS, AND VOLUMES.

80. *English.*

1 mile = 1760 yards = 5280 feet = 63,360 inches.

1 square mile = 3,097,600 square yards = 27,878,400 square feet.

1 cubic yard = 27 cubic feet = 46,656 cubic inches.

Metric.

1 metre = 10 decimetres = 100 centimetres = 1000 millimetres.

1 square metre = 100 square decimetres = 10,000 square centimetres = 1,000,000 square millimetres.

1 cubic metre = 1000 cubic decimetres = 1,000,000 cubic centimetres = 1,000,000,000 cubic millimetres.

COMPARATIVE TABLE OF ENGLISH AND METRIC MEASURES.

1 metre	= 39·37 inches.
1 square metre	= 1·196 square yard.
1 litre	= 1·76 pint.

GENERAL TABLE, SHOWING RELATION BETWEEN LENGTHS, AREAS, AND VOLUMES.

81. *Lengths and Areas.*

A square, the side of which has a units of length, contains a^2 units of area.

A rectangle, the sides of which have a and b units of length, contains ab units of area.

A triangle, of which the base is a and the vertical height b units of length, contains $\frac{1}{2}ab$ units of area.

A circle, the radius of which has r units of length, contains πr^2 units of area.[1]

An ellipse, the axes of which contain a and b units of length, contains $\pi\frac{ab}{4}$ units of area.[2]

A cube, the edge of which has a units of length, is enclosed by $6a^2$ units of area.

A sphere, the radius of which has r units of length, is enclosed by $4\pi r^2$ units of area.

Lengths and Volumes.

A cube, the edge of which has a units of length, contains $a \times a \times a$ units of volume.

A rectangular bar, of which the edges are respectively a, b, and c units of length, contains $a \times b \times c$ units of volume.

A circular cylinder, the height and radius of which have h and r units of length respectively, contains $\pi r^2 h$ units of volume.

[1] The value of π is about 3·1416. It does not vary, as it expresses always the number of times the circumference of any circle is longer than its diameter. The radius is usually denoted by r.

[2] The correspondence with the circle is seen if we remember that $\frac{a}{2}$ and $\frac{b}{2}$ will take the place of r. Hence instead of $\pi \times r \times r$ we have $\pi \times \frac{a}{2} \times \frac{b}{2}$ or $\pi\frac{ab}{4}$.

A sphere, the radius of which has r **units of length**, contains $\frac{4}{3}\pi r^3$ **units of volume.**

(It may be noticed that a circular cylinder into which a sphere will just fit is just half as big again as the sphere. Take $h=2r$, then cylinder is $2\pi r^3$, and 2 is half as big again as $\frac{4}{3}$.)

NOTE.—It is strongly recommended that a number of exercises should be worked out with a view to making the quantities in the above tables as real as possible. It is one thing to know a table of measures by heart; it is quite another thing to understand them, and to be able to form a mental picture, such as will exhibit relationships between the quantities themselves and familiar objects around us. In order to gain this real knowledge of standards of measurement, efforts must be made to reproduce them as frequently as possible, and one convenient mode of so doing is to draw lines and areas to scale corresponding with real lengths, areas, and volumes. Examples follow:

1. Draw lines corresponding with a mile and a kilometre. Use any scale.
2. Cut out in paper any two areas which are related in the same way as an acre and a square mile.
3. Draw lines showing the relation between a cubic metre and a cubic decimetre, and between a cubic millimetre and a cubic centimetre, etc.

SUMMARY OF ARGUMENT IN MEASUREMENT OF SPACE.

82. In the preceding section **measurements have grown more complex** than those which were made at the beginning of our course. Our first exercises in measurement were of the simplest class, namely, of length and mass.

Now it is quite clear that measurement of length or of mass may become, *under some circumstances*, far from easy. There may be special difficulties in individual cases. But the *reasoning* required, and the *methods* employed, are, in their nature, very simple.

The methods employed in the measurement of area and of volume do not differ in anything essential from those by which length and mass are measured. In the first place, it is necessary to select a standard by which to measure, and **without which measurement can never be made.** There are various standards in existence, but the **units** which form part of the general and scientific system of measurement are the **square centimetre** for area, and the **cubic centimetre** for volume.

By the comparison of these units with the quantities of area or volume to be measured, we obtain **numerical values** just as in the case of length and mass. We learn again the same lesson, *that quantity, of whatever kind, must be estimated by means of another quantity of the same kind.* An area is so many times another area; a volume is so many times another volume. Further, areas may be added together or subtracted from one another, and the same operations may be carried on in the case of volumes by means of their numerical values. **Nothing more than this can be said with regard to their quantity or magnitude.**

83. But there is a difference *in the nature of the quantities themselves*, and it is that difference which renders their measurement more complex than previous measurements. An area is more complex than a length, and a volume is more complex than an area. It is possible to regard an area as constituted of or containing *two or more lengths.* It is also possible to regard a volume as constituted of or containing *at least three lengths.*

We regard area or surface as having **two dimensions** (length and breadth), and volume or space as having **three dimensions** (length, breadth, and thickness).

Area and volume are forms or portions of space, and on this ground the chapter has the heading of Measurement of Space. The dimensions mentioned need to be measured *at right angles* to one another.[1]

In order to acquire anything like accurate knowledge of space under the form of area or surface, it is necessary to understand what is the nature of **a plane surface.** In learning about a plane surface, we have also learnt what is **a straight line.**

84. The meaning of a plane surface is best learnt by practical experience. Plane surfaces are the only surfaces known to us of which *any two* fit, or come in contact at all points. Two surfaces fitting *may be* of any shape. We learn very little from finding out that *one* surface fits another, but when we can procure **three surfaces any pair of which will fit,** then these surfaces must be of a certain kind, and they are said to be **plane.** *None but plane surfaces obey this condition.*

Straight lines resemble planes in this respect, that straight lines alone can be made to coincide at all points throughout their common length. The derivation of our knowledge of a straight line from that of a plane explains this matter.

The practical employment of these views of planes and straight lines surrounds them with great importance.

A test of a similar kind may be applied to determine the nature of certain angles. Right angles are those angles which are able to form *a straight line*

[1] This statement must be accepted as the most suitable one for the beginner.

with two of their sides, when two of them are placed back to back in the same plane; or which entirely occupy the surface around a given point, when four of them are placed in the same plane to coincide with this point.

85. Both area and volume may possess an attribute called *shape.* Shape is independent of quantity, that is, the knowledge of the *shape* of an area or of a volume does not imply a knowledge of its *magnitude.* **An area or a volume may be changed in shape without altering in numerical value. Shape is not a quantity.**

EXPLANATION OF SOME OF THE TERMS USED IN THE PRECEDING SECTION.

86. Space.—Space, as well as matter, is made known to us by the senses of touch and sight. By these senses we learn to distinguish between *matter* and the *space in which matter exists.* Matter is said to occupy space. When we observe portions of matter as distinct bodies, we must necessarily perceive that *something* in which they exist, and which forms a separation between them. That *something* in which all the objects we can perceive, including the earth, sun, stars, etc., exist and move, is called space. Since we are unable to perceive material objects without being aware of that which is not material—namely, space—we may rightly say that we learn about matter and space through the same sources. *The two ideas enter the mind at the same time.* We may measure lengths, areas, and volumes in space. That is, we may measure lengths in *one, two, or three directions at the same time.* Yet it must always be remembered, *that none of these measurements can be carried on in the absence of matter to mark the beginning and end of each operation.*

Volume.—Such a portion of space as exhibits length, breadth, and thickness. Any portion of matter must occupy a certain portion of space, and when we speak of the magnitude of this portion, we call it the *volume* of the body. The same idea is conveyed by the words *size* and *bulk*. In describing the internal volume of a hollow body or vessel, the word *capacity* is used. Bodies of different shapes may have the same volume, and vessels of different shapes may have the same capacity.

SHAPE.—The arrangement, relative position, or disposition of the parts of a body, constitutes its shape.

SURFACE.—Bodies are separated from one another or from space by one or more surfaces. For example, a sphere is separated from space by one continuous surface. A cube is separated from space by six distinct surfaces. A surface is an extension of space, over which one body ends while another body or space begins to exist. A surface cannot from its nature possess thickness, but it must possess length and breadth; that is, it has two dimensions only. The magnitude of a surface is called its *area*.

LINE.—The boundaries or edges of surfaces are lines. Or, the separation of one surface from another, or of two portions of the same surface, is effected by a line. The division of any two plane surfaces from one another is effected by a *straight line*. A line whether straight or not has only *length*. It cannot of necessity have either breadth or thickness by its definition.

RELATION.—This word is frequently used with a special meaning to denote comparative magnitude. The relation between quantities is found out by comparing them by means of a standard. Hence, relation is indicated by numbers or *numerical values*, and it will be perceived that relation in this sense can only exist between quantities of the same kind. When we speak of relative magnitude or relative quantity, the same idea of comparison is conveyed. The relation existing between two numbers or quantities is called their *ratio*. It is known that different numbers may denote the same quantities, according to the standards which have been used to compare them, but these numbers must be always in *the same ratio*.

EXERCISES IN MEASURING AREA AND VOLUME.

N.B.—*An account of each experiment must be carefully written out in a note-book. Wherever possible, drawings must be introduced.*

1. Draw a square with each side 10 cm. in length, and after calculating the number of squares with sides of 1 cm. which it will contain, proceed to draw these.

2. As an exercise in neatness, divide a square centimetre into square millimetres.

3. Draw any figure, not a square, which has an area of one square centimetre.

4. How many square millimetres are there in a rectangular surface which is 3 by 2 cm. (3 cm. long and 2 cm. broad)?

5. Prove by the use of paper and scissors that a rectangle 3 by 2 cm. has the same area as a parallelogram 3 cm. long and 2 cm. in vertical height.

6. Cut out of cardboard a square having a side of 4 cm., after drawing it carefully, and find its mass; then find the mass of a circular piece of cardboard having a radius of 3 cm. The result will enable you to calculate the number of square centimetres in the circular surface of the cardboard, provided the mass of the cardboard is evenly distributed. (The number found should be $3 \times 3 \times 3{\cdot}1416$, or πr^2 sq. cm.)

7. Measure the total area of a box (use a box of weights) in square centimetres. How many square millimetres of surface would there be?

8. Measure the volume of the same box in cubic centimetres. How many cubic millimetres of space does the box fill?

9. Cut out pieces of paper to completely cover the box of weights, and rule upon each surface lines to represent the number of square centimetres of surface. (Neglect any fraction of a centimetre in the linear dimensions.) Or, tie pieces of white thread in each direction across it at intervals of 1 cm. You will then be able to realize the number of cubic centimetres contained, and you will see that the value for the area of any object is always different from that expressing the volume.

The same result may be obtained by drawing surfaces, equal to those of the box, upon squared paper.[1]

10. Measure the area of the floor in square feet, and then calculate from your result the cost of whitewashing the ceiling at 2d. the square foot. What length of paper 1 foot wide would entirely cover the ceiling? (It may be reckoned that a length is *used* if it has to be cut narrower to fit.)

11. What fraction of an acre is the area of the floor? An acre is 4840 square yards. (That is, how many times is the area of the floor contained in an acre?)

12. Draw a square which is 8 cm. in the side. Draw lines to represent the number of square centimetres contained. Then inscribe a circle of 4 cm. radius, and sum up the number of square centimetres in the circle approximately, calculating the fractions of square centimetres as accurately as possible. Compare the result with that obtained by adding together the number of square centimetres *not* enclosed by the circle, and subtracting them from the whole number within the square. Compare both results with that obtained from the statement-area of circle $= \pi r^2$.

13. Measure the total surface of a rectangular block with a rectangular opening in the centre. If such a block is not to hand, measure the total surface of a drawer. Measure also the total volume of the wood in a drawer.

14. Ascertain the area of an irregular piece of paper by tracing its outline upon a piece of "squared" paper (*i.e.*, paper ruled into squares for scientific purposes), or upon a piece of paper which you have yourself divided into square centimetres. It will be necessary to add together the squares and the fractions of squares as correctly as possible. Note the advantage of thickening every fifth line each way, and so obtaining large squares holding 25 small ones. The calculation becomes simpler and less liable to error.

[1] Paper ruled into square centimetres, and also into square millimetres, will often be required in the Laboratory, and is purchased from the ordinary sources, but in an observation of this elementary character it is recommended that paper should be ruled into square centimetres by the observer himself.

15. How many square centimetres of paper are there in the given book? How many square centimetres of surface does this paper possess?

16. Measure the volume of the interior of a drawer.

17. Find the capacity (or internal volume) of the bottle, by filling it with water, and pouring the water it contains into a graduated vessel, such as a graduated glass cylinder. Make several observations and take the mean result as correct.

18. The litre (a measure of volume which abroad takes the place of our pint or quart, being equal to 1·76 pint), is equal to 1000 c.c. How many litres would there be in a cubic metre?

19. Find out the volume of a body by displacement of water in the following ways:—(1) Take a plain glass tube which has a narrow end, fitted with an india-rubber tube and a clip, as on a burette, to allow water to run out when required. Pour in water to fill about half way, then mark the level with paper or thread. Place the body in the water, and run out the water into a measuring vessel until it sinks to the original level. We now have run out water equal to the volume of the body immersed. (2) Perform the same experiment, but use a graduated tube (a burette) instead of a plain one, and notice the first level, and the level of the water after the body is immersed. The change of level denotes the volume of the object, for the tube has been graduated so that a certain volume corresponds with a given length along the tube.

20. Run out 10 c.c. of water from a burette into a weighed beaker, then on weighing again, the mass of the water may be found, and that of 1 c.c. may be calculated. And make another measurement by running 1 c.c. into a counterpoised watch-glass in order to confirm your result. Then assuming that all the water you use has its mass quite evenly distributed, or, in other words, that every c.c. of water has the same mass as that which you have just measured, you may find out the volume of a given quantity of water by ascertaining its mass. (The assumption made is a very necessary one, for we have so far no grounds for taking so much for granted.) You may then find out the *volume* of a body by finding the *mass of the water which it displaces.* Instead of learning its volume directly from

graduated vessels, we can learn *indirectly* by the use of the balance.

21. The converse of the above exercise may be performed. Find the mass of 1 c.c. of water by measuring the mass of the water displaced by a body of known volume.

By a similar process measure the mass of a volume of water equal to five times the volume of the solid provided. (Note the level of water in a burette, immerse the solid, run out liquid until the previous level is reached, and repeat the operation four times.)

22. Counterpoise a beaker on the balance, and then weigh a given body by finding out the volume of the water, delivered from a burette into the beaker, which will counterpoise it.

23. Find the volume of several small flasks up to a certain mark by weighing the water they contain. These flasks may be marked and set aside for future use, if the value can be relied on.

24. Find out the volumes of water required to raise by 1 cm. the levels of water contained in a large beaker and in the narrow neck of a flask. Also measure the volume corresponding with a difference of level of 1 cm. in a large graduated cylinder, and in a fine burette.

From each of these observations you may calculate the relative areas of the surfaces of water. You will also learn that the *smaller the surface* from which measurement is made, the *greater the chance of accuracy* in the result. You will learn in addition *that the calibration marks on vessels which measure volume are not linear measures, although the volume may be proportional to the change of level, and so to a linear distance.*

25. Measure the volume of air in a flask standing over water. The flask should be about half full. First measure by marking the level of the water by means of gummed paper together with a pencil mark, and then filling the emptied flask with water to the same mark. This water may be poured into a graduated cylinder. In the next place, take another measurement by placing the thumb under the mouth of the flask, and then inverting it. The volume of water now required to fill up the flask, to the place occupied by the thumb, may be ascertained by graduated vessels, and will give a measure of the air. Which is likely to be the more accurate method?

26. Compare the volumes of the two given bodies by finding the change of level produced by each when it is immersed in water contained in a tube or vessel of even bore (that is, with the same diameter or cross-section). The lengths which denote the change of level will be proportional to the volumes of the bodies.

27. A pipe has a cross-section of 9 sq. cm.; how many grams of water will there be in a metre length, assuming 1 c.c. of water to be 1 gram? It must be remembered that unit of area × unit of length = unit of volume, or, in other words, volume is measured by the area of a cross-section multiplied by the number of units of length through which that cross-section is maintained without change.[1]

28. By measuring the dimensions (diameter and length) of the graduated portion of the given burette, and then calculating the volume, ascertain if the volume agrees with what is represented on the burette.

29. A circular cylinder 25 cm. in height holds 1 litre; what is the internal diameter? Find out first what the cross-section must be; then calculate from this the diameter, knowing that $\pi \times r^2$ = area of circle. Make some practical measurements of the height and diameter of graduated glass cylinders, and demonstrate the accuracy of the statement that *height × area = volume.*

30. Draw a large number of radii at equal intervals in a circle of about 8 cm. diameter. Treat the circle as now divided into a known number of triangles, and multiply the area of one triangle by the number present. The area of the circle will then be nearly the same as is obtained from πr^2. (That πr^2 can be proved to be the area, may be demonstrated by division of the circle into a large number of triangles—see Fig. 44.)

[1] That the numerical value of volume may be derived from multiplying the numerical value of the area of the cross-section by the numerical value of the length, in the case of a body of constant section, can only be realized by building up figures by the use of little cubes. For this purpose cubes of boxwood roughly filed down to 1 c.c. are very useful.

31. Find the total area of a circular cylinder. Demonstrate your method by covering the cylinder with paper, and calculating its area; and also by rolling the cylinder along a flat sheet of paper so that a rectangular area is obtained. Note that you have as a result two circles, and a rectangle of which the sides are respectively the height of the cylinder and the circumference of its section.

32. It is required to compare the diameters of two tubes indirectly. Close the end of each securely by a cork. Now pour into each the same volume of water. The heights of the columns will now be *inversely* proportional to the diameters, but not *simply* proportional. We must take the square roots of the numbers representing the heights, and the double of the numbers obtained will correspond to the diameters. This calculation is based upon the method of finding the volume of a circular cylinder. To find the volume of such a cylinder, we multiply the area of the base, or πr^2, by the height. Now the volumes of the two cylinders are alike; therefore $\pi r^2 h = \Pi R^2 H$, and h is related to H (the two heights) as ΠR^2 is related to πr^2 or as R^2 is related to r^2.

33. Demonstrate that a triangle has an area equal to half that of a rectangle on the same base and of the same altitude.

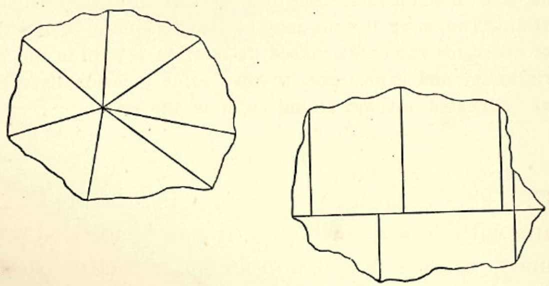


FIG. 55.—Illustration of methods of measuring irregular areas.

This may be done by cutting out surfaces in paper as shown in Fig. 42, and referring to Euclid. Knowing then how to find the area of a triangle, we may find the area of any irregular figure, such as that above, by dividing it into a number of

triangles. The same area may also be measured by taking measurements as shown above, perpendiculars being taken from a diameter and a series of figures being formed, the areas of which can be calculated.

Draw an irregular figure on paper, and calculate its areas by both methods here given, and also by tracing the same figure on squared paper.

34. Ascertain by immersion in water, contained in a graduated vessel, the relation between the volume of a sphere and that of a circular cylinder, of which the diameter and height are each equal to the diameter of the sphere. Perform a similar experiment with a cube and a sphere, and refer to the relations expressed in the table of section 81.

35. What will be the volume of a cube which will have the same total surface as the top of the bench?

36. What must be the *diameter* of a circular cylindrical vessel in which 1000 c.c. of water stand at a height of 25 cm.?

PRACTICAL DETAILS AND SUGGESTIONS IN TEACHING.

NOTE.—Inasmuch as the method is more important than the means, the instruments described in several instances are not essential. The same lessons may be learnt equally well with the aid of other instruments; indeed, it is often a gain in the matter of originality and experience, to turn aside occasionally from the course suggested without losing sight of the end.

MEASURES.

87. There is no little advantage in starting measurements with objects which are not graduated in the ordinary sense. For example, it is better to make the measurements of small lengths by using a strip of paper an inch long. A number of such pieces of paper may be cut at the same time with scissors for the whole class. After these have been used a straight lath of a foot in length may be used, and this may be subdivided when necessary with the aid of the inch strip of paper. The advantage of a graduated measure is more appreciated after following this course.

Many exercises in copying and comparing measures which were not worth introducing into the text will suggest themselves. Short exercises of this character are often needed to provide occupation for the quicker workers at the end of a lesson.

The steel measures of a foot long, which are graduated in inches and fractions on one side and in centimetres and millimetres on the other, are of the greatest service. They will also stand rough usage. Any ironmonger will provide them at about fifteen shillings the dozen.

A few metre measures made of boxwood will be needed, and one good standard metre should be available for reference.

Note that the end graduations of a scale are often worn and not to be trusted.

It is recommended also that pieces of brass should be let into the work-benches throughout the laboratory to denote distances of a metre. The distances of a metre, decimetre, and centimetre, and also of a yard, foot, and inch, should also be conspicuously shown in one or two places in the laboratory by thick lines painted on the walls and plainly named. These aids will be found very valuable in training the eye to remember dimensions.

To these may be added with advantage a conspicuous representation of a square decimetre and a square foot.

A certain number of graduated cylindrical vessels and flasks will be needed. A few litre flasks and

a few litre cylinders will suffice; but a larger number of 250 c.c. cylinders will be needed, and also a number of small graduated flasks, for example those of 25, 50, and 100 c.c., which may be weighed on the balance.

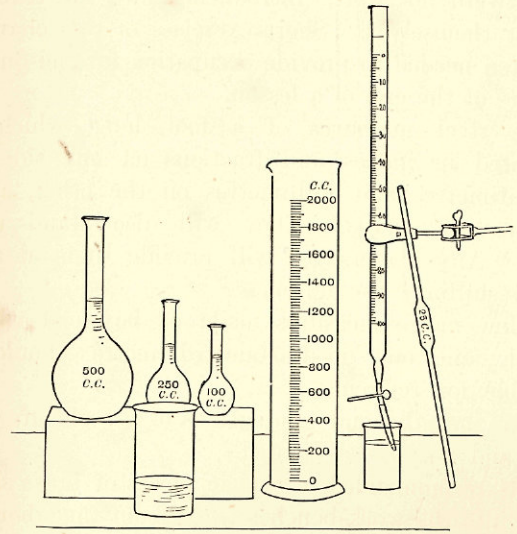


FIG. 56.—Vessels used in measurement of volume.

Burettes with a caoutchouc tube, and either a clip, or a piece of glass rod fitted inside the tube, are to be preferred for elementary work to those provided with a glass tap. White paper placed behind aids the reading of level. But the usual precautions in working with a burette must be pointed out, namely: (1) The burette must be upright. (2) The measurement must be always made from the centre of the lower line of the curve of the liquid surface. (3) The delivery-tube must stand full of liquid during measurement; and (4) the eye must be on a level with the surface in reading. The use of floats may be encouraged.

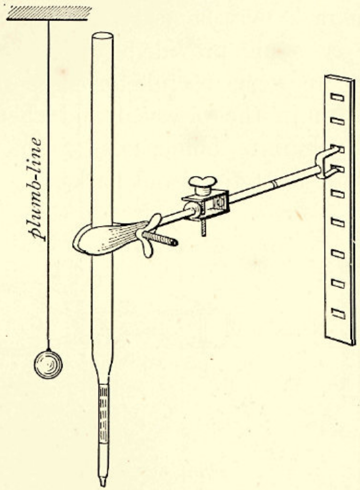


FIG. 57.—A burette fixed correctly to give accurate readings.

Clamps and supports are a constant source of trouble unless they are well made. I have found my own models stand the test of time admirably, but they are somewhat expensive. The basis of the design is a patent brass rack,[1] shown in Fig. 58. Into this fits at any required height the end of the burette holder, which is made of such shape as will readily permit it to be placed in or taken from the rack, which may be easily screwed against the wall or an upright where required. The other end is constructed in the ordinary fashion to grip the burette by means of a screw. This portion is connected with

[1] Supplied by Mr. Tonk of Birmingham.

the other by a ball and socket joint permitting variation in the position of the vessel supported. A dozen of these racks, about 8 inches in length, are fastened against the wall above a long bench.

The same rack may be used for supporting a ring attached to a stem provided with the same end. These rings are very useful for supporting beakers and flasks both in the physical and chemical laboratory, and the burette holder also forms a very convenient support for retorts and flasks.

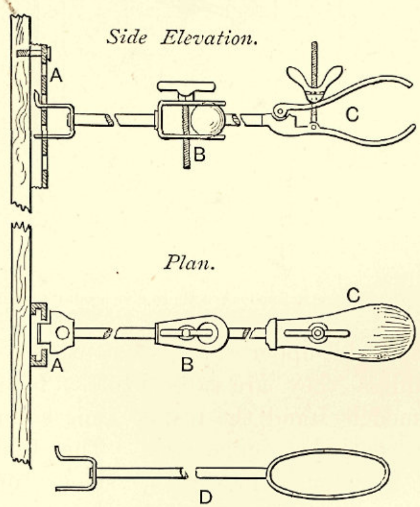


FIG. 58.—A form of holder for burettes, flasks, etc., in brass. *A* is a rack; *B*, the ball and socket joint (the sockets are steel); *C*, the clip. *D* shows the same rack utilized for a ring support.

It is advisable to perform a large number of exercises in comparing the capacities of vessels, and in testing the graduations already marked on vessels, not forgetting that some vessels are made to *deliver* certain volumes while others are made to *contain* the volumes denoted. Burettes are for delivering volumes of liquid, while flasks are seldom used to deliver required volumes.

MASSES.

88. For weighing by means of an indefinite standard wood and lead are useful, on account of the readiness with which they can be cut with a knife.

For adjusting and making up masses, small-sized shot may be used, but it has certain disadvantages for junior classes. Nails, odd stoppers, etc., serve for irregular masses.

The spiral which is used for measuring mass is best made in the workshop. One end of a thin brass or german-silver wire is fastened to a cylinder held in the lathe. The lathe is slowly turned, and the wire strained and guided by hand into a spiral on the cylinder. Various thicknesses of wire will give varying rigidity to the spring.[1]

For standard masses, grams and multiples in brass should be used. For many purposes, pounds and multiples of pounds will be found useful. They are easily obtained in sets from the ironmonger. The weights in pounds with rings attached must alone be purchased, for these weights will nearly always have to be suspended. If fractions of a gram are obtained, those made of *aluminium* are much to be preferred. Small fractions are seldom required, and should therefore be seldom served out to junior forms. Hence it is advisable to procure separate boxes of grams and

[1] It may be pointed out here, that well fitted metal workshops and a skilled mechanical assistant constitute the necessary foundation on which a physical laboratory should be built. The laboratory should grow out of the workshop in many senses,

of fractions of the gram. The purchase of fractions may be saved by buying aluminium foil and having the various fractions constructed and measured by the classes. Their value may be impressed on them by number dies, and decimal values of the gram are less likely to mislead than the value in milligrams, which is sometimes given by the manufacturers.

Various forms of spring balances will be required, and may be obtained through any ironmonger. Those which are suspended when in use should be obtained, as they will be needed afterwards in various exercises in mechanics. A set graduated in fractions of an ounce, and a number graduated in pounds will be required. Some of the forms of letter balances will serve to display how weighing may be performed without the use of the ordinary beam balance. Such a balance is shown in Fig. 13. Modifications may easily be devised and graduated.

BALANCES.

89. The structure of balances makes it necessary that they should be used with great care. They should be periodically inspected, and all dirt removed. The knife-edges and planes, when made of steel, must be kept free from rust. For physical observations the large open balances illustrated (Fig. 14) are very suitable, and are readily adapted for weighing bodies in liquids. It must be remembered that large masses tend to strain the beam, and sudden or uneven movements may diminish the accuracy of the suspension. For this reason the beam should be placed in suspension with a steady movement, and it should be stopped from swinging when the pointer is in the centre of the scale. Of course, alteration of the masses in the pans must only take place when the balance is at rest and supported. Before using a balance it should be dusted, if necessary, and made to swing, in order to test its accuracy of adjustment. If it swings equally on each side of the scale, or if it swings nearly equally, it is ready for use. It is better to *allow* for a little inequality of swing than to constantly alter the adjustment. Sometimes it is difficult to get the balance to swing. In that case, blow one of the pans very gently. Most substances should not be allowed to touch the pans for fear of injury to them. A filter paper on each pan is advisable for direct weighing.

Finally, great care is needed in supervising the manipulation of weights, and in seeing that they are properly used, and properly returned to their right position in the box after every operation. No excuse should ever be admitted for neglect of this important rule. In weighing, the weights should be counted when in the pan, and also when they are returned to their places. Before using the weights, it is advisable for each student to add together all the weights in the set, giving to the smaller weights their correct decimal values. By exercises in selecting the weights needed to make up a given mass he must make himself familiar with their values.

Rolls of wood on which ribbons have been wrapped may be obtained from drapers and will serve for various measurements, *e.g.*, measuring the relation of the circumference to the diameter of a circle, and for measuring the volume of a cylinder.

A large number of small cubes are very useful in conveying the relations which exist between cubes of various sizes. These may be cheaply obtained of any

size from a saw-mill, or through a toy merchant. A certain number of cubes of boxwood may be obtained by filing to the required size. They are useful for exhibiting the volume of 1 c.c.

The precautions which are needed in measuring such dimensions as the diameter of a sphere or cylinder are best shown practically. The right use of straight edges, and the way to turn a scale so as to get the graduations in touch with the point to be measured; the need of placing the eye on the same level with, or straight in front of, the object to be measured; and other lessons, constitute no small share of the learner's work.

As the error in reading the position of a pointer against a scale is less, the further the eye is from the scale, it is often useful to observe the pointer from a distance through a telescope. Another method adopted in some instruments is to have a mirror behind the scale, and place the eye so that the image of the pointer is hidden completely by the pointer itself.

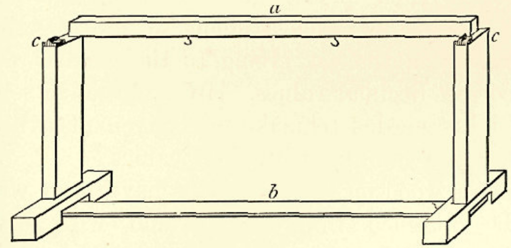


Fig. 59.—A useful form of stand for various purposes.

A very useful form of stand, which is capable of serving a great variety of purposes, is here described. It is about 3 ft. 6 in. long, and from 18 to 20 inches in height. The cross-bar of oak (*a*, Fig. 59) is

$1\frac{1}{2} \times 1$ in. in section, and is let into the middle of two solid deal uprights 5×2 in. in section, which are supported on feet tied together by a bar (b), which should not be placed at the centre of the feet, as it would then be in the way of objects suspended from the oak bar. Brass hooks at intervals are screwed into the bar, and cast-iron knife-edges (c)[1] are screwed on the top of the uprights, two on the one side of the bar at right angles to the bar, and two on the other side of the bar and parallel to it. For exercises in weighing masses by springs, the principle of the lever, for observations with an inclined plane, for testing strength of rods and many other purposes, I have found this form of stand very useful. It has the advantage of enabling the work-benches to be kept clear for ordinary work, and renders permanent fittings unnecessary.

[1] These are very cheaply cast, but old triangular files will serve when cut up.

CHAPTER VI.

DIRECTION AND POSITION IN SPACE.

FURTHER MEANING OF PLANE AND ANGLE.

90. The meaning of planes and plane figures has been given in a previous section. It now remains to be proved that such ideas as these enter largely into the practical work of the world; not only into the construction of machinery and measurements of all kinds, but also into all our thoughts and knowledge of **direction** and **position** in space. Unless we are certain as to the meaning of a plane, not to mention a straight line, there is little chance of our understanding how to describe a position, or how to find a position when it is described.

Although it is not probable that many people will trouble themselves to understand completely what constitutes a plane, yet most people have a fairly accurate knowledge for practical purposes. This is perhaps all that may be demanded of them.

In the same way you may be able to *draw* a circle with a pair of compasses without being able to give a *definition in words* of that figure. Yet the operation you perform, if properly described, does accurately define the result. And the method of preparing a plane surface, as we saw (in sect. 75), is nothing but a *practical rendering of the definition of a plane.*

A circle is a figure enclosed by a line, which is made by a point being traced on a plane, at a fixed and unaltered distance from another point, until it returns to the position of starting. The compasses enable this to be done, because the pointed ends remain the same distance apart. The same operation may be carried out less accurately, by using any object such as a piece of string to keep a tracing point, such as a pencil, at the same distance from the stationary point.

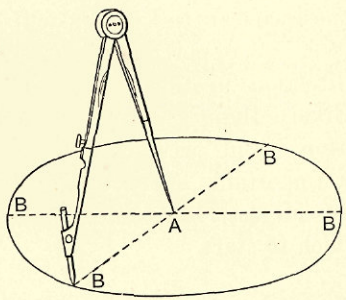


FIG. 60.—The operation of tracing a circle, the distance *AB* remaining constant.

In carrying out this operation we have done something more than to describe a circle. A curved line has been drawn on a plane in such a way, that every possible point, which lies in the plane at a given distance from the given fixed point, is included within this curved line. In fact, there is no point which is within a certain distance from this fixed point and not within the circle. The circle entirely covers a certain portion of the plane. The given fixed point is called

the *centre*, and the curved line the *circumference* of the circle.

91. Now it must be remembered that a plane may be supposed to extend to any distance. The material plane surface, which has been described in the last section and of which you have learnt to prepare a specimen, coincides with an imaginary plane surface extending without limit in all directions. The plane of the paper on which a figure is drawn does not *end with the paper. The plane itself has no boundaries.* We may therefore draw, or imagine to be drawn, any number of circles, with circumferences extending further and further away from the given point. Now any one of these concentric[1] circles will give us, as we shall soon see, an opportunity of learning what is meant by direction in the plane; although nothing seems less likely than a circle to give information about direction. But we shall learn at the same time a very important lesson, **that direction cannot be ascertained nor described, unless there be some fixed point from which to start.**

92. Let us suppose a straight line *AB* to pass through the centre of the circle which has been drawn, and be terminated by the circumference. Such a line is called a diameter. Of these straight lines there may be any number; and for our purpose any one may be selected.

It will be found, however, that in proceeding to draw such a line, we draw it from *left* to *right* as below (Fig. 61). This is merely a matter of convenience, the words *left* and *right* referring to our

[1] *Concentric*, having the same centre.

position with regard to the paper. It is the easiest direction in which to draw, when sitting in front of the paper. **The words right and left refer to the person drawing the line, and give no information to any one else, not even to his neighbour unless the paper is undisturbed, as to which of the many lines passing through the centre it is.** And even as regards the person drawing the line, the description of right or left carries with it no very definite meaning. There is no exact position denoted by the word right.

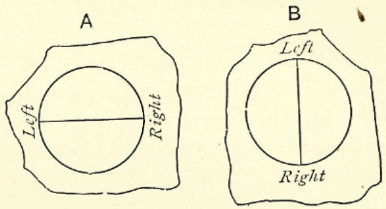


FIG. 61.—An illustration of the fact that the words *right* and *left* have no meaning in themselves, but must be taken with regard to something else. The piece of paper, on which a circle with its diameter has been drawn, gives different meanings to those words, in the two positions represented by *A* and *B*.

But having selected for ourselves one of the diameters, we may now draw another at right angles to it, and we shall have divided the circumference into four equal parts, or quadrants. Since these diameters, by their intersection, make four right angles, we learn the fraction of the circumference *subtended*[1] *by* a right angle. The whole circumference of any circle is by custom divided into *360 equal divisions* or *degrees*, and the measure of a **right angle will always be 90** of these degrees, or as it is written, 90°. **An angle of 1° becomes the unit angle, the standard to measure by; and we find out**

[1] *To subtend* is to stretch over and include.

the magnitude of an angle by seeing how many times it contains the unit angle, just as we have measured other quantities.

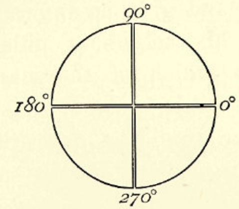


FIG. 62.—The four quadrants of the same circle, each containing 90 degrees, and fitting together.

93. Now it is clear, that if we take any four right angles, and place them together in any way, we shall find the whole of the surface around their meeting point completely occupied. **There will be no unoccupied surface where they meet.** This follows from what has been said, and demonstrated practically, in a previous

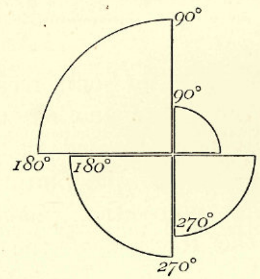


FIG. 63.—Four quadrants of different circles fitting together.

section about the meaning of a right angle; and it agrees with what we can here perceive to be the relation between a right angle and the quadrant of a circle. For did not the construction of the circle imply, *that every portion of the plane within its cir-*

circumference had been enclosed by the circle? The tracing point never left the plane, but took in succession every possible position on the plane, which was within a certain distance of the fixed point. But we have not yet learnt what all this has to do with direction or position. The reply is that we are laying the foundation of the general meaning of position, by learning to describe position within a circle. We must begin by understanding about angles.

94. Starting with a circle of which a diameter has been drawn, we can *imagine a point travelling from one extremity of the diameter, round the circumference, until it reaches the other extremity.* It will have passed exactly *midway* the point from which a straight line

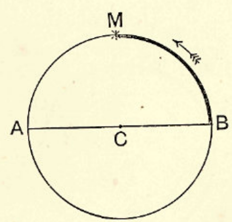


FIG. 64.—A point travelling round the circumference of a circle has reached a position at *M*, which is the same distance from either extremity of the diameter. A line joining *M* with the centre of the circle, *C*, is perpendicular to the diameter.

drawn to the centre will make two right angles with the diameter (four angles are formed if the line passes through the centre). Proceeding further the point *will at last return to its starting place.* That is, we have followed out the process of constructing a circle.

Instead of imagining a point to be travelling over the circumference, imagine the **radius of the circle**, that is, a straight line with one extremity at the centre of the circle and the other at the circumference, **to be**

carried round with its central end fixed. (A practical illustration of the process is displayed by the movement of the string, fixed at one end and carrying a pencil at the other end, by means of which it was shown that a circle could be described.) Then we shall have **a series of angles,** growing bigger and bigger, *formed between the original position of the string or line and its position at various moments.*

95. Hence it will be clearly understood that the **size of the angle** depends on nothing else but **the extent of this rotation.** *The length of the line sweeping out the angle has nothing to do with its size.*

It is quite easy to imagine the line with which we are dealing to be very much longer. But the size of the angle made by the rotation of a longer line is

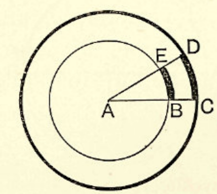


FIG. 65.—The same angle is formed by a point *B* travelling on one circle, and a point *C* on a larger circle. But the distances passed over by these points are not alike.

not thereby increased. It is as well for every one at this stage to draw on paper a series of concentric[1] circles, and it will be perceived that the **same fraction of the circumference is subtended by the same angle, whether the circle be large or small.**

This is shown in Fig. 65, where the point *B* travels over the distance *BE* in forming the angle

[1] Having the same centre.

BAE, but the point *C* on the larger circle has to pass over a longer distance *CD* to form precisely the same angle.

Actual length of line gives no indication of the value of an angle, but extent of rotation does. *Hence we always learn the size of an angle when it is said to be of so many degrees.* **The number of degrees tells us how far the originating line has been rotated,** and this number is independent of the length of the lines enclosing the angle.

96. It will be as well to remember that we do not generally consider two straight lines which are in the same straight line (that is, *two parts* of the *same* straight line) as forming an angle of 180°. And we do not generally speak of angles which are larger than 180°, as we may judge them by what remains to complete the rotation; for example, an angle 270° may be described as an angle of 90° (for $360-270=90$). Yet there are a number of instances in which angles

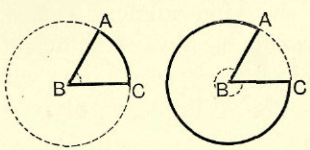


FIG. 66.—The angle *ABC* may be regarded as 60°, or as 300°, according to circumstances.

must be regularly measured from 0° to 360°, and in this case **it must be remembered that an angle of 60° may have been traced out by a point travelling over 300°, as well as by one travelling over 60°.** This is shown above (Fig. 66). It is usual to consider the tracing point as always travelling in *the opposite direction to that of the hands of a watch.*

SUMMARY.

97. A circle is a figure made by a line traced on a plane at a constant distance from a fixed point. The fixed point is called the centre, and the curved line traced out is called the circumference. This curved line encloses every point on the plane within a certain distance from the centre. Still we have no means at present of describing the position of any one of those points, nor of distinguishing one point on the circumference from another. Position has no meaning for us so far. The words *right* and *left, top* and *bottom* give no information, unless they are taken in connection with something which is fixed. The first step to be taken is the division of the circumference of any circle into 360°. Then it is shown that the value or magnitude of an angle is dependent on the distance through which the tracing *line* has been turned away from the line at rest, and not at all upon the length of these lines. Hence it follows that the same fraction of the circumference of any circle is subtended by the same angle. Hence, also, the magnitude of an angle is always sufficiently described by the number of degrees of a circle which it subtends. A right angle, for example, is always 90°. And we may regard any angle as measurable by means of the unit angle of 1°. Angles may therefore be *compared* with one another, and they are quantities.

EXERCISES IN MEASUREMENT OF ANGLES.

N.B.—*Each of the following experiments should be carefully carried out. Some of them form valuable lessons in practical geometry.*

1. Draw angles of 90°, 45°, 30°, and 15°.
2. Draw angles of 135°, 150°, and 165°

3. Draw angles of 235°, 270°, and 345°.

4. Show by construction that all the angles in an equilateral triangle are equal. Construct an angle of 60° by the intersection of two equal-sized filter papers, so that the circumference of each comes in contact with the centre of the other.

5. Show also that the angle of 90° is $1\frac{1}{2}$ times that of 60°.

6. Draw a clock-face on paper, and state how many degrees correspond with the space of a minute. Then construct a clock-face on a larger scale, by describing a larger circle from the same centre, and drawing straight lines from this centre through the minute marks of the smaller face to meet the larger circle.

7. What is the angle formed by the two hands of a watch at 12, 24, 36, and 48 minutes past 12 o'clock? Do not forget to take into account the movement of the short hand. Draw diagrams of each.

8. The long hand of a watch is 2 cm. long. How long will it take the extremity to travel over 50 metres?

9. Compare the distances moved through in equal times by two points on a wheel, one of them being 3 and the other 4 inches from the centre of the axle. Draw a diagram to illustrate that the comparative distances passed through by the points are the same as their distances from the axle.

10. Construct a right angle, then make the two straight lines forming it equal to 3 and 4 respectively, taking any convenient

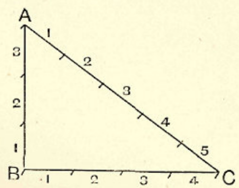


Fig. 67.—The two sides of the right angle ABC are 4 and 3 times a unit length. Then the distance from A to C is 5 times the same unit.

lengths as units. Then prove by measurement that the straight line joining the extremities of the two lines will measure 5 on the same scale (Fig. 67).

11. Make use of this discovery to construct a right angle.

Measure off on a piece of string consecutive lengths in the proportion of 3, 4, and 5; mark the lengths by pieces of thread; then arrange three pins so that the string, when stretched and bent so as to have its two ends at the same point, shall have its marked points touching the pins. We then have a triangle with one of its angles a right angle. (The proof of this relation existing between the sides of a right-angled triangle is contained in prop. 47, Book I. of Euclid, in the shape of the squares on the two sides containing the right angle being together equal to that on the third side. The numerical equivalent of this proposition is $3^2+4^2=5^2$.)

12. Cut out a triangle in paper, and show, by cutting off the corners, that its three angles are equal to two right angles. Repeat your observation with other triangles to show it is always true. Prove also, by cutting out paper, that the alternate angles made by two straight lines are equal.

13. Fix 3, 4, and 5 pins upright in the bench, so as to be in the same straight line, by using your eyesight. Test your result afterwards by a straight-edge.

14. Place pins at the ends of two unequal laths. Then arrange these laths, while keeping them parallel, so that the pins at each extremity are in a straight line with another pin. Find out by several observations the relation between the lengths of the laths and the sides of the triangles of which they form part, as shown in Fig. 68.

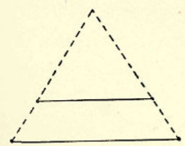


Fig. 68.

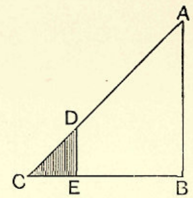


Fig. 69.

15. Cut out a square of paper from a given rectangular sheet, and fold it across a diagonal; that is, divide it into two right-angled triangles, each having its right angle contained by two equal sides. The other angles of the triangles will be each equal to 45°. (See Fig. 69.)

Make use of one of these triangles to find a given distance AB, marked by pins on the bench-top and supposed to be inaccessible. Move the paper, while keeping DE parallel to AB as far as possible, about until the sides CD and CE of the triangle are in the same straight line with A and B respectively. This may be done by looking along the edge of the paper. If we can now make sure that the angle ABC is a right angle, then the distance CB, which can be measured, will be equal to AB, which has been assumed to be inaccessible. (The inaccessibility may be made real by placing some object between A and B.)

It is clear that the triangle ABC is *similar in shape* to the paper CDE,[1] and therefore the side CB is equal to AB. Test your result by actual measurement of AB.

16. Make use of the same figure to ascertain the height of the room, holding the paper vertically, and looking along the edge CD. When the paper has been moved until CD points to the ceiling line, measure the distance of C from the wall. It will be necessary to fix the paper at such a height that the eye can be placed at C, and *this height must, of course, be added to the distance from the wall* to obtain the height of the room. (Caution is needed to maintain CE horizontal, or the result will be inaccurate.)

17. Measure the distance between two points A and B which do not permit direct measurement; for example, the distance between two pins at the edges of two benches. Place another pin in one of the benches at C, so that ABC may form a right angle; then place another pin at D, in the same straight line with C and B, and at the same distance from C as B is found to be. Then find a position E in a line drawn from the point D at right angles with DCB, such that a pin placed there may be in the same straight line with the pins at C and A. The distance DE, which can be measured, will be the same as distance BA. Test your result by measuring the length of a string stretched across from B to A. (See Fig. 70.)

[1] The similarity in shape follows from the angle DCE being common to the two triangles, for since both of the angles DEC and ABC are right angles, the remaining angles BAC and EDC must be each 45° and equal. Hence the side CB must be related in magnitude to the side AB, in the same way as the sides CE and ED are related.

Construct the same figure on paper, cut out the triangles ABC and CDE, and satisfy yourself that they are equal. Prove also that they are equal by the aid of Euclid.[1]

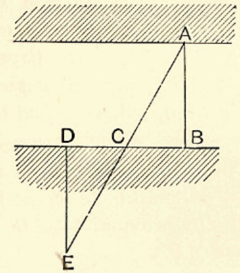


FIG. 70.

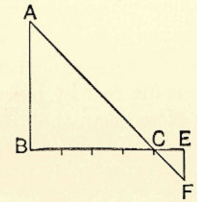


FIG. 71.

18. Place two pins at any convenient distance from the rectangular corner of the bench, as at A and B (Fig. 72). Place

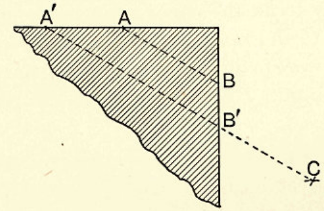


FIG. 72.

another pin at C. Then gradually move the pins by equal steps, as measured along the sides of the bench, until they are in the same straight line with C. We have moved the line AB

[1] An extension of this method is sometimes of great service. In Fig. 71 AB may be a distance across a river which needs to be known. Now, it may not be possible to walk back far enough along EF to get your eye in line with a stake fixed at C and some object at A. It might be necessary to walk half a mile, and inequalities of ground or other causes might interfere. In that case, make the distance of BC much greater, fix a stake to mark the point at C, place another stake at a distance CE one quarter of BC, and then walk backwards until your eye at F is in a straight line with C and A. The distance EF will then be a quarter the width of the river. A further extension of this method should be attempted.

(or rather the direction AB) through a given distance, and it has remained parallel to its original position.

19. At the end of a rectangular bench are points B and C. It is required to find the distance of the point A which is in a straight line with B and C. Make CD equal to CB, and from the points B and D measure off equal distances BF and DE of such length that A, F, and E are in the same straight line. Then $BA = BF$ or DE.

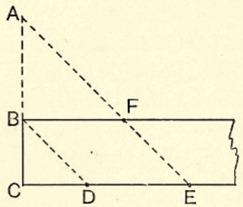


Fig. 73.

Satisfy yourself that BA does equal BF, both by actual measurement and by geometrical knowledge derived from Euclid. Join BD for the latter purpose.

20. Cause the light, from a candle or gas behind a screen with a small opening, to be reflected from a plane mirror so as to appear on the wall. Let the mirror be supported so as to be capable of turning. Mark the position of the bright spot on the wall. Turn the mirror through a measurable angle; mark the position of the spot of light; and measure the angle through which the reflected light has been turned, by finding the angle made by the two positions of the spot with the surface of the mirror from which the light was reflected. Take several observations, and show that the angle moved through by the mirror is half that moved through by the spot of light.

21. Draw on paper all angles at intervals of 5° from 0° to 90° in a given quadrant, and measure the relation of the perpendicular, let fall from the extremity of the radius, to the radius itself. Draw up a table with the lengths of these lines in terms of the radius, and compare them with the values given in a table of Natural Sines.

22. By using squared paper show that vertical height may be calculated by a horizontal distance and an angle. Construct a quadrant of a circle of considerable radius on the paper, and draw angles from 5° up to 85° at intervals of 5°. Produce the lines in each case until they meet a line drawn vertically from the base line at its extremity. The vertical heights corresponding with each angle may then be read off.

23. Make use of the property of a mirror, described above, for finding the height of the room. Obtain a horizontal reflection from the mirror placed vertically. Turn the mirror until the spot of light reflected by it from a luminous body touches the top of the wall. Then the angle made by the light before and after reflection is known,[1] and the distance of the mirror from the wall is measurable. Now it can be arranged that this angle shall be 45°, by moving the position of the mirror. This makes the horizontal and vertical sides of the imaginary triangle equal. (But this adjustment need not be carried out by those who can calculate from the value of the tangent, nor need it be done in cases where graphic methods are understood.)

24. Having placed a mirror at the same level as a light, cause the light to be reflected until the spot appears *at the edge of the ceiling*. Then intercept the light by a piece of paper held vertically at some little distance from the mirror. Mark the spot, and so obtain the vertical height corresponding with the angle made by the movement of the light at a shorter distance away. The two heights, that on the paper and that on the wall, will be in the same ratio as the horizontal distances of the mirror from the paper and from the wall respectively.

25. Measure a base line AB, and make a pin at the edge of a board coincide with A, then place pins in the board in directions AB and AC, C being a distant object not in line AB. Draw angle BAC; place a pin in the line AB at the edge of the board, and make it coincide with B, and move the pin which at first coincided with A into the line BA; place a pin in the line BC, draw angle ACB. Move the line in direction of BC, parallel to itself, till it intersects the line in direction AC at D, and intersects AB at E. Then the relation of AB to AE is the

[1] Being twice the angle through which the mirror turns.

same as the relation of AC to AD and BC to BE, or $AB : AE = AC : AD = BC : ED$.

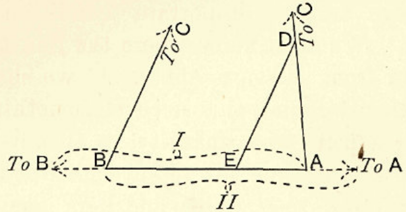


FIG. 74.—The diagram shows the lines drawn on the board when the observations are completed.

26. Taking it for granted, that when the image, in a flat mirror, of a stretched string is in the same straight line with the string itself, the string is perpendicular to the mirror, find out the angle between two mirrors by measuring the angle made by two strings stretched in front of them.

THE MEANING OF DIRECTION AND POSITION IN A GIVEN PLANE.

98. We have learnt in the previous section what is meant by an angle, and also how we measure the size of an angle. It has been shown that the **magnitude of an angle** is determined by *the extent to which one line of the angle is turned away from the other*, while their two extremities remain coincident and fixed. And it has been shown that the line which has traced an angle denoted by 360°, that is, an angle which has been made by a complete circular sweep, must have passed over every single point within the circle.

Any given point or position *within the circle* must have been passed over by the line at some period or other in its journey. We need to know the stage at which any point was passed over, if we wish to learn anything of its **position.** The angle formed at that

stage must be found out, and this will tell us something of position.

But there is one important condition not yet mentioned. **We must know where the line tracing the angle starts from.** Again and again we shall find, as we have already found, this need of **something to start from, either a point or a quantity,** in all measurements. Length, mass, and time all require for their measurement *something to start from;* and here again, in determining position, the same limitation is prominent. We know nothing whatever of the position of any point on our plane figure, although we may know that *at some stage or other* we must have passed over it in sweeping around with our line. *Some point must be considered as fixed,* then we can proceed to measure from it.

99. If we fix upon the point A in Fig. 75, and say that a given point within the circle *makes, with that point A and the centre of the circle, an angle of* 60°, all that is needed is to carry round the tracing line BA until, at the position BC, it makes an angle of 60° with its original position. *Then the point required must be somewhere or other in this line.* **Whereabouts in it we have no means of knowing at present.**

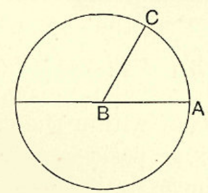


FIG. 75.

Now the statement, that the point is in a line which makes an angle of 60° with the line AB, does not give sufficiently full information about it. It is true we assumed that it was somewhere *within the circle described.* **But then the circle may be imagined of any size; just as the lines** BA **and** BC **may be of any length, though always containing an angle of 60°.** It must dawn upon our minds then, that the *angle* made with any fixed point in a given plane is *not enough* for description of position. We should have to *grope our way,* as it were, *along the line* which forms the given angle, until we found the point.

To avoid this search for the point, we should have to state the distance along the line from the centre of the circle where it could be found. **We then have the two necessary kinds of information about position in a plane,** viz.:

1. **Direction.**
2. **Distance in that direction.**

100. It will be perceived that a result precisely similar to this might be obtained by taking *any two fixed points* as the basis of our description of an unknown position. We have had, in one sense, to do so

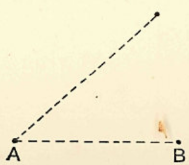


Fig. 76.

already, for in the circle the *centre* needed to be known or we could not have drawn the given angle. But if we have two points A and B fixed, we can join them and then trace out from the line drawn AB an angle of the described magnitude. We must next

measure the right linear distance along the tracing line. (Fig. 76.)

We learn then the same lesson from each method—the need of **direction and distance to define position.** Either *a circle with a fixed and known point from which to measure angles,* or *a line with a fixed and known extremity from which to measure angles* will serve the purpose.

It will be advisable at this stage to make use of these means of describing position in the following exercises.

EXERCISES IN DESCRIBING POSITION.

N.B.—*In writing out a description of these exercises, take care to point out how each one illustrates the need of knowing two fixed points and a distance, before position can be determined.*

1. Select two points A and B on your bench; find a point C which is 25 cm. from A, and also makes a right angle with the straight line joining AB. (Fig. 77.)

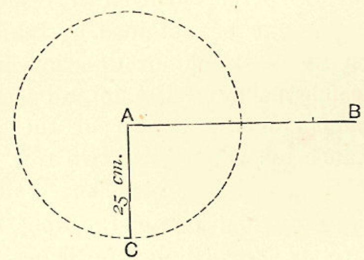


Fig. 77.

2. Find a point C which is 50 cm. distant from a point A, and 45° away from a point B placed at a metre distance from A.

3. Find the angle made by two points A and B with a third C, the three points being marked on the bench. Use for the purpose a circle drawn on cardboard and divided into degrees, each ten degrees being marked by longer strokes. A lath with a straight edge will serve for pointing out direction.

The same observation should be taken with two points on the wall, to show *that we may observe angles in any plane.*

4. Place a pin upright at the distance of 1 metre from another pin so as to make an angle of 60° with a given object. Make use of the divided circle constructed in question 3. Devise a means of testing the accuracy of your result without referring to the divided circle.

5. Draw a circle and its diameter on paper, and show that there are four points on its circumference which are equidistant from the diameter. Show that they are at the same time equidistant from a diameter drawn at right angles to the first one. Note then that distance from one or even from two diameters does not effectively describe position.

6. Make use of the divided circle again to measure the angle between a line and a point on the bench, and move the point until it makes an angle of 45° from each end of the line. Test your position by finding if the distance of the point from each of the extremities of the line is the same. Also find a position in which the point makes an angle of 60° with the line. What more is needed to construct an equilateral triangle?

SUMMARY.

101. When two straight lines meet, an angle is formed. The magnitude of the angle has been shown not to depend upon the length of these lines, nor upon the distance by which their two free extremities are separated from one another; but it does depend upon the extent to which one of the lines, *as a whole,* would need to be turned before being sufficiently separated from the other to form the given angle. A circle, described with its centre at the meeting point of the lines forming the angle, affords the best means of describing this magnitude. The circle may be of any size. Its circumference is divided into 360 equal divisions, and the unit angle subtends one of these divisions.

The **direction** in which any point lies in a plane may be described by constructing a circle on that plane with another point as centre. We proceed next to find at what position on the circumference of the circle a rotating line (with one extremity fixed at the centre) would, if long enough, coincide with the point of which the direction is to be described. The position of this line on the circumference can, however, be described only with reference to some other position, which must be taken as the starting point of the rotation. The number of degrees of the circle between these two positions adequately describes the direction required. In describing direction in a given plane, it is essential, therefore, to have two points already known: one from which to draw a straight line indicating the direction, and the other to mark the angle through which the line has been turned. The **position** of the point investigated is indicated by its distance away from that fixed point, which is at the extremity of the line showing direction. **Without two points being known, neither direction nor position can be described.**

POSITION IN ANY PLANE.

102. It is not difficult to perceive that there is **another important requirement** for ascertaining position. We may have positions *above* or *below a given plane.* There are positions in all planes, and some additional means of finding them is needed.

It was stated in a previous section (No. 60), that a sphere may be traced out by the rotation of a circle about its diameter. Fix upon any line passing through the centre of a circle and then, keeping its two extremities fixed, the circle may be considered to turn round upon these points, and also upon the line joining them. That is, not only do these points remain fixed, **but the whole diameter remains fixed, and at rest, while the circle is made to trace out a sphere.**

A penny made to spin on the table illustrates the meaning of this, for when moving rapidly it produces somewhat the appearance of a sphere.

When we imagine a given sphere to have been formed in this way, we shall at once understand **that every point on its surface is equidistant from its centre,** which is the same as that of the circle from which it is derived.

And since the circumference of a circle, rotating about any of its innumerable diameters, always sweeps out *the same spherical surface,* it is clear that **when cut in any plane passing through the centre, the surface presented by the segments is a circle equal to that of the generating circle.** (When cut by a plane not passing through the centre smaller circles are produced, Fig. 78.)

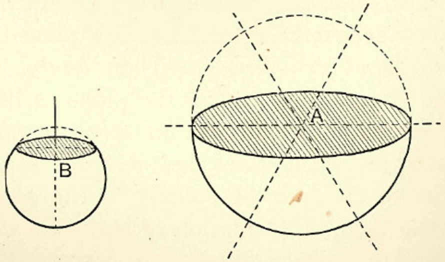


FIG. 78.—*A*, a circle generating a sphere. *B*, a sphere so cut as to present a smaller circle than that which generates it.

103. Now a little consideration will show, that just as the circle in a plane is formed by the rotation of a line, which sweeps over all the surface within its circumference; and just as its tracing line has therefore pointed during its revolution in every conceivable direction in that particular plane; so does the sphere **include every possible plane in space,** in that it can be generated by the rotation of a circle about any diameter. An unlimited number of planes pass through its centre, for there is no end to the number of positions to be taken by the generating circle.

Both the circle and the sphere may be looked on as being **a portion of an indefinite whole,** the *circle* standing for the *limitless plane* in which it exists, and the *sphere* for the *whole limitless space* in which all things exist.

104. It has been shown that position in the circle may be defined by a direction and a distance (or angle and distance), provided we have a fixed point from which to start. So much for position in a given plane. But suppose the plane itself be not known, then this must be first determined. The knowledge already gained with regard to position will make it clear, that we can only **determine the position of a plane** by having some *fixed plane for reference.* Then, *having found the plane,* find the position within the plane as before.

It is evident that position in space is not easy to define, and that there are *two distinct stages* in the complexity of the subject. It is by the *use of angles* that we can define the relation of *one plane to another* with regard to position, just as we can in the case of lines. In Fig. 79 the angle *ABC* defines the relation of the planes to one another. It will be interesting to take as a problem the discovery of the condition requiring to be observed, before the lines give a true indication of the relation of the planes with regard to position. (Note that *relation of position* is the only relation which is possible between planes; there is *no relation of magnitude*, for example.)

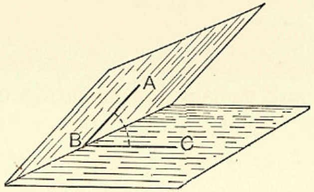


FIG. 79.—The angle *ABC* shows the relative position of the two planes which contain the lines *AB* and *BC*.

ANOTHER METHOD OF DESCRIBING POSITION.

105. We have learnt from preceding lessons that we know nothing of the position of bodies, except by referring to some other bodies which we look upon as fixed or unaltered in position. Whether they are *really fixed or not* makes no difference, so long as we know them to be **fixed with regard to the body under observation.** It is of little concern to us whether the earth is fixed or not in space, when we are trying to give a description of the position of London on its surface; nor is it important to know whether a table rotates with the earth, when we say that a book is lying at a certain place on the table. London is fixed on the surface of the earth, and the book is fixed with regard to the table. On these surfaces we must take some *points or lines,* which are fixed thereon, in order to define the position of London or of the book.

We have already learnt, that even when we know the plane on which exists a point to be determined, we must have some lines or fixed points for the purpose of measurement. Now the simplest description will be that of a point upon a plane which we consider to be

known. As a matter of fact, *our most frequent experiences* in finding position are upon a known surface, that of the earth, which may be looked upon, *roughly* and *for small areas,* as a plane.

After describing position on a given surface we may proceed to position in space. Now in the first case, the simplest one, **two measurements alone are needed.** In the second case, **three at least are required.** For example, *all points on the circumference of a circle* are equidistant from its centre, so *one* measurement is not enough to distinguish a point; whereas *all points on the surface of a sphere,* which covers all directions in space, are also equidistant from its centre. **One linear distance does not suffice for position on a plane, nor do two linear distances give information about positions in space.**

106. To take the more common requirement, that of position on a known plane (a small surface of the earth for example), once given a fixed point, we may draw two straight lines meeting at right angles at this point.

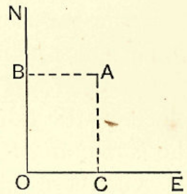


FIG. 80.—The position of the point *A* with regard to *ON* and *OE* is defined by the length of the lines *AB* and *AC*.

Then we can measure *linear distances from these lines* to the given point, provided it is included within the angle (Fig. 80). If it be not, the straight lines *can be produced;* or, if need be, they can be continued so as *to intersect* (Fig. 81), and then the required point

must be within one of the four right angles so formed. Provided we *know the angle* within which the point is included, its **linear or shortest distance from each of these lines fully and adequately describes its position.**

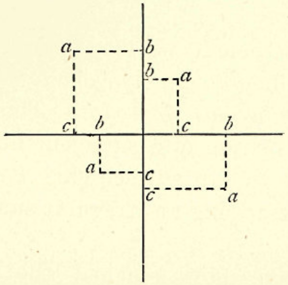


FIG. 81.—The position of the point a in each of the four quadrants is defined by the lengths of the lines ab and ac.

These distances describe the position of the point with regard to two straight lines which are at right angles and pass through the fixed and known point.

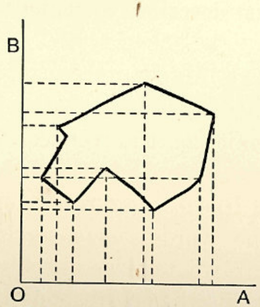


FIG. 82.—A method of describing an irregular surface, by stating the distances of various points on its boundary from two fixed lines OA and OB.

It may be observed that we have taken as our problem the determination of the position of a **point** (or we may rather say of a *particle*, which is a body

so small that its dimensions may be neglected). The position of a **body** may be determined by the same process, by treating it as a **particle**. But if this be not convenient, we may take the position of *any particle on its surface*, and from this learn the position of the whole.

From this statement we may learn what may have seemed very difficult, namely, *how to describe the shape of an irregular surface or body.* This may be done by taking the position of a sufficient number of points on its boundary line or surface (Fig. 82). **This is the only means of describing an irregular surface.**

107. We have now reached the stage to fully understand why it is needful to have points, **North, South, East,** and **West,** for the purpose of describing position. These are names which denote *a system of defining position,* which has been in use from the remotest ages, and which is one and the same thing as **the method just described of using two lines at right angles.**

If we take two lines intersecting at right angles, and put *north* at the upper extremity, and *south* at the lower extremity of the vertical line, then *east* at the right extremity, and *west* at the left extremity of the horizontal line: we represent on our paper, what is meant by the words *north, south, east,* and *west.* Or, *better still,* if we take a sheet of squared paper, and consider that the corresponding extremities of *any* of its vertical and horizontal lines are similarly named, we shall gather the meaning of these words (Fig. 83).

It must be remembered, of course, that the points marked *on the paper* do nothing more than indicate *direction with regard to the paper.* Corresponding directions are imagined on the surface of the earth, although what we may have called north on the paper may be quite the opposite of the real north. In fact, we have done nothing more by our diagrams than to show the *relative directions* of N., S., E., and W.

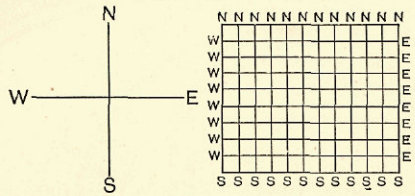


FIG. 83.—The direction of North, South, East, and West, shown in two ways. It will be seen that the *points* denoted by these words vary with the position of the speaker, whereas the *directions* do not change.

108. We next come to the meaning of *real north.* Some point, it is admitted, must be considered as fixed for the starting-point of all measurement. Such a point there is on the earth, and it is called the north. Any point on the surface of the earth would serve as a starting-point, but **unless we could constantly refer to that point,** or be able at any moment to state where it lies, measurement and determination of position would be as impossible as ever. Fortunately the direction in which the point lies, which has been selected and called the north, can be found at any time without difficulty.

A magnet, if balanced on a point or placed so that it can easily turn in any direction, will always point in the same direction, provided no iron be near it. Why it does so need not be explained at present. A **compass** consists of a magnet so balanced that it turns

in this direction, *i.e.*, the north.[1] It will generally be found to have a divided circle upon which are marked N., S., E., W., and intermediate points.

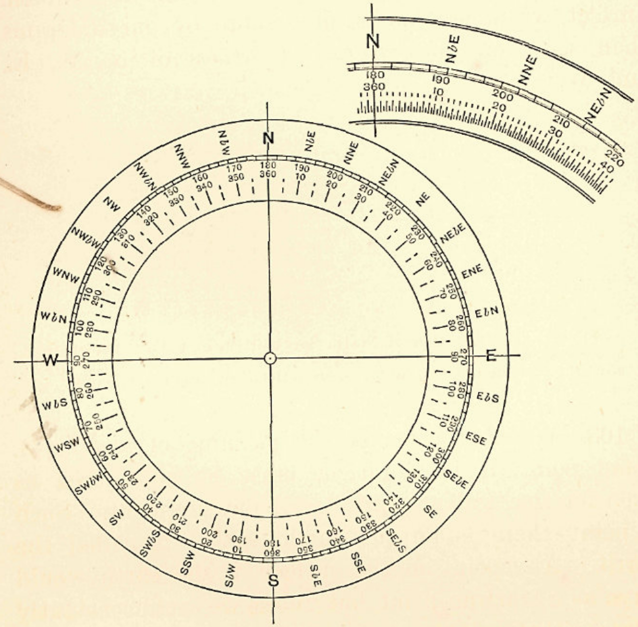


FIG. 84.—A graduated circle showing the points of the compass. A small portion alone of the circle is graduated in half-degrees.

The principle upon which these intermediate points are named will be best learnt by making a copy on paper of the divisions of the circle on a compass with

[1] It may be as well to point out here, that the direction called north does not exactly agree with that indicated by the compass. Somewhat complex considerations have to be taken into account if a more accurate knowledge of the **standard direction** is required. The need of distinguishing between the *position*, north, and the *direction*, north, is likewise important.

their names (Fig. 84). **It must not be supposed that the N. on the circle, which is fixed, shows the direction of the north.** The letters on the circle point out, for the sake of convenience and rapidity of calculation, what *a given direction would be in case the N. of the circle were the real north.* Some additional meaning is given to the word north when you try to realize that, were you at the North Pole, you could only travel away from it by going south. Whichever direction you took you would move southwards.

109. In addition to the magnet as a means of showing the north, we may always rely upon **the sun at 12 o'clock**, or **the Pole Star** at night. At 12 o'clock or mid-day, the sun is in the south,[1] and we shall be looking to the north if we stand with *the sun at our back.* Throughout the night time the Pole Star points out the north, remaining fixed in position while other stars are in apparent motion.

Having fixed upon the north, we can now obtain lines at right angles to one another, one going from north to south, the other from east to west, and distances from each of these lines will be an adequate guide to position.

MEASUREMENT OF POSITION ON THE SURFACE OF THE EARTH.

110. The method described in the last sections needs some *modification,* if it is to be applied to the description of position on the **surface of the earth.** In dealing with a perfectly plane surface, all that has been found necessary is to measure from *two*

[1] With reference to countries north of the Tropics.

straight lines; and it is most *convenient* that they should be *at right angles* to one another. But in dealing with such *large distances* as become necessary in treating of the surface of the earth and the position of towns, mountains, rivers, etc., on its surface, we have *two difficulties* to face: first, **the fact that the surface of the earth is not flat but spherical;** and, secondly, **the practical difficulty of comparing position by referring to two lines alone.** Even although they may be fixed in any agreed position, the *numerical values* of the distances would be **so large** as to cause great inconvenience.

The first difficulty need not be dwelt upon, for it is easy to substitute (in the case of the earth) for our *two lines* at right angles, *two circles*[1] lying on the surface of the earth in planes at right angles to one another, as shown (Fig. 85).

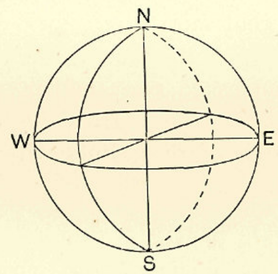


FIG. 85.—The surface of the earth divided by two circles, at right angles to one another, for the purpose of describing position.

But in connection with this scheme it must be remembered, that there are *an indefinite number of such circles,* which may all with equal truth be said to be at right angles to one another. It is necessary to fix upon *one* to start with as the *standard.* There

[1] Circles approximately.

is no difficulty in this if we know of **any fixed point.** We have already learnt that the magnet is always able to point towards the north, and we shall soon learn that a *more accurate indication* of true north is possible. Hence we may select that line which, passing through a known spot (the *Observatory at Greenwich* for example), points in the direction of the north. The imaginary line, passing through the Observatory at Greenwich and running north and south, is the **standard line** *for measurement towards the east or west.* It is called the *Meridian.*

111. It now becomes necessary to define the position of the other **standard line.** But the method by which this is done; and the manner in which the standard north and south is more accurately determined, is better understood after a statement has been made in regard to the earth's movement in space. A brief description of this movement must suffice.

The earth moves round the sun once a year, and it spins as it goes. A body spins on an axis, an imaginary line around which turn all the parts of the body except those in the axis itself. The spinning or *rotation* will account for **day and night,** provided it do not take place with its axis always turned towards the sun as in No. 1, Fig. 86, in which case one portion of the earth would be *always dark,* and the other *always light.*

If the axis were turned, as in No. 2, and maintained always in that position with regard to the sun, then there would be **equal day and night all over the face of the earth.**

The direction in which the axis is actually fixed is shown in Fig. 87, but instead of remaining in that

position always *with regard to the sun,* **the axis is directed always to the same point in space.** It is directed in fact towards the **Pole Star.** Consequently,

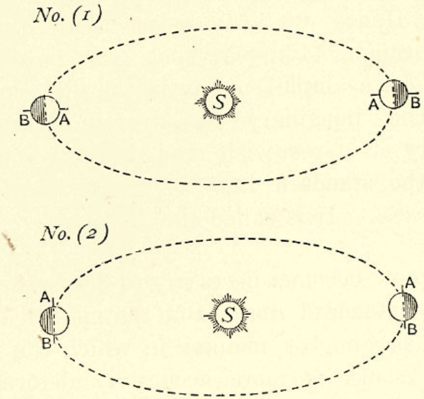


FIG. 86.—If the axis upon which the earth rotates be turned towards the sun as shown in No. 1, the surface of the earth around *A* would be always light, and that around *B* always dark. If the axis be placed as in No. 2, then every portion of the earth would possess equal day and night.

it assumes with regard to the sun the various directions shown in Fig. 87 at different periods of its annual journey round the sun.

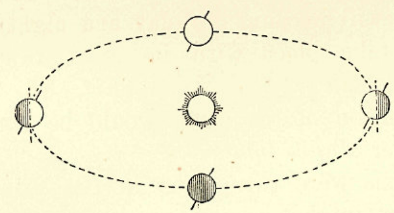


FIG. 87.—The actual direction of the earth's axis being fixed with regard to a point many times more distant than the sun, and that direction being inclined at an angle to the plane in which lie the earth and the sun, then different portions of the earth's surface are *directly* exposed to the sun at different stages of its annual progress round the sun. It will be summer over a given surface at that period of the year during which the surface is most directly exposed to the sun, and on account of the daily movement of the earth, that surface will form a belt round the earth.

From this cause a given part of the earth's surface **is differently inclined to the sun at different periods of the year.** When placed so that the sun shines *most directly* upon it, that part is passing through the season of **summer.** When placed so that the sun shines *more obliquely* than at any other time, that part is passing through **winter.**

112. Now it is observations of the movement of the earth with regard to the Pole Star, which inform us of the direction in which the *axis* lies. The terminations of the axis are the **Poles,** north and south. **In these two poles we have the fixed points which render measurement of, and description of position upon, the earth's surface possible.** By means of a magnet, an approximate north (the *magnetic north*) alone is obtained; whereas the axis of the earth gives the direction of *true north.*

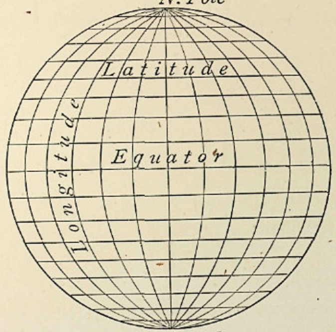


FIG. 88.—An illustration of the division of the earth's surface by circles of latitude and longitude.

From these two points a series of parallel circles are imagined to divide the surface of the earth into

zones, in much the same way as the circle is divided into degrees. They are called **Parallels of Latitude.** That which is midway between the poles is called the **Equator.** Counting from the equator to the north and south pole respectively, there are 90° of latitude.

A careful study of an atlas will now help us to understand, that in the parallels of latitude just described, together with circles passing through the poles which are called **Meridians of Longitude,** we have all that is necessary to describe any position on the earth's surface, if we follow the method already described. Taking the **meridian of Greenwich as the standard,** there are 180° east, and 180° west, marked by meridians. It will be perceived that as we go towards the north or south, the value of a degree of longitude diminishes.

The convenience of this system of marking the whole surface of the earth with lines, *distinguished by numbers,* is extremely great. We have always, within convenient distance, two lines at right angles, with which the position of any point can be compared.[1]

SUMMARY.

113. Besides the method already stated for describing position in a given plane, it is always necessary to know exactly the position of the plane itself before any description can be complete. Not only may the point sought be in any plane, but the position on the

[1] To make the preceding description more intelligible, it is strongly recommended that small globes fixed on a stand should be used to demonstrate the reason of the seasons. Cheap papier-mâché globes can be obtained, and a pointed wire can be inserted to hold them in the right position. It is worth while devoting some time to the explanation and demonstration of this subject by means of models and diagrams.

plane may be unknown. There are two stages in the search for position. In order to determine a position which is completely unknown, at least *three* linear measurements have to be made; and these measurements should be at right angles to one another.[1]

The description of position in a known plane may be most conveniently effected from two lines at right angles; and this is the method which is followed, as closely as is possible on a curved surface, in describing the position of places on the surface of the earth. The only difference between them lies in the fact that all the lines of latitude and longitude are *curved* instead of straight. Any map or chart, with its lines of latitude and longitude, illustrates this method. We learn that in finding position, just as in measuring any kind of quantity, we need a *standard* from which to start. **The standards in Geography are true north and the meridian of Greenwich.**

EXERCISES IN DEFINITION OF POSITION.

1. Draw a plan of a path which proceeds for 4 miles straight to the N.W., then 5 miles to the W., then 2 miles due N. Use the scale of an inch to the mile.

2. On the same scale, an inch to the mile, draw a path leading from a given point for 3 miles in a straight line to the E., and then 4 miles due N. till it reaches another point; find out the nearest distance between these two points by measuring the distance on your plan.

3. Find the points of the compass by means of a magnet, and draw a plan of the room which shall show its true position when the top of the paper is assumed to be the north.

[1] We may substitute for these three linear measurements the measurement of an angle and of one linear distance, but the plane of the angle must be known.

4. A magnet is hidden in a piece of wood. Find out the direction in which the magnet lies without uncovering it. To do so, allow the piece of wood to be suspended or balanced freely, and ask which is the north.

5. Ascertain by the use of an atlas and a magnet the direction of London, Paris, and New York with regard to the room.

6. Focus a telescope on a given movable object, then take its position in the room by measurement from two walls and the floor. Remove the object and replace it from the measurements taken. The telescope will serve to test your accuracy.

It will be quite clear from this example that any position in space may be defined with regard to a given plane. A plane is considered as fixed; in the case given, the floor is naturally selected, and then any point marked out on this plane by measuring distances from two lines at right angles on the plane, the lines made by two walls at right angles for example. From the point thus determined, distance is measured vertically from the plane. If the position of the point is unknown, its vertical distance from a given plane is measured; and the position of the point to which measurement is made may be ascertained on the plane, by the ordinary course of taking the distance from any two lines at right angles.

7. Project a circle on a plane placed at an angle with it. This is done by a plumb-line being carried round the edge of the circle. (One made of cardboard, or the disc of an electrophorus will serve. Support it by a clamp.) The successive positions of the plumb will trace out an ellipse. Vary the position of the circle, and note the alteration in the shape of the ellipse.

8. Draw a clock face and put in against each figure the point of the compass corresponding to it, assuming that the line joining XII. and VI. is the N. and S. line.

9. How many points of the compass does a minute on the clock face represent?

10. If a compass card were used for a clock face, where would the minute hand be at 17 minutes past the hour?

11. Determine the angle between two lines, by reading the position of a compass needle when the N.S. line of the card is made to coincide with each in turn.

12. Compare the direction of the compass needle with that of the shadow of a vertical stick at twelve o'clock, or the given direction of true N.; transfer this given direction to another part of the room, and notice whether the direction marked by the compass still agrees with it.

13. Use the compass to transfer a line parallel to itself.

14. Measure by means of a compass the angle turned through by a cupboard door.

15. Draw a plan showing the laboratory, without details, and the position, relative to it, of the rest of the school buildings, and the *direction* of the various passages and paths leading away from the laboratory.

16. You are required to cover the surface of a sphere *uniformly* with a simple geometrical design, so that the figures used may be all of the same size. Mention a few figures which may be used.

CHAPTER VII.

MEASUREMENT OF TIME.

THE MEANING OF TIME.

114. We may gain some knowledge of **time** by comparing it in our thoughts with **space**. It will often be found that much of a subject is learnt by a process of comparison. Even when the things compared are unlike in their nature, and although different subjects ought to be kept distinct in our minds, there may be *some aspects* in which they are alike. Yet this process of comparison would not be helpful except on one condition, namely, that the qualities in which the likeness may appear should be **more familiar to us, more commonly presented to our minds.** In that case the less familiar ideas will become more intelligible. They will be more readily grasped by being linked together in our minds with those which are better known.

115. It is not difficult to understand that when we perceive that the bodies around us, tables, chairs, books, and the rest, are *distinct* and *separate*, we begin to feel the need of **some idea to express this separate-**

ness of objects. The meaning we give to the term *space* expresses this. We cannot think of the objects familiar to us except as existing in space. It cannot be said that matter depends *in* any way on space, but it can be said that the idea of space presents itself as soon as we perceive separateness.

The *extent* of this separateness we have measured under the name of *length*. We have also learnt that lengths and measurements of space can only be measured when we have some fixed point to start from. Hence we do not know anything about space, at any rate in a scientific sense, *except by means of matter*. And if we start by assuming we know something about matter, we need to link the notion of space with our thoughts about matter. **We recognize space as that necessary something in which matter is perceived, and through which separate and individual portions of matter exist.**

116. Now in just the same way we perceive *events and occurrences to be distinct and separate from one another*. An event is happening now. Another did happen. I saw a dog. I see a man. The wind blew. The tree fell. These statements, or rather the occurrences which are the foundation of the statements, *carry with them, quite apart from their difference of nature or kind, the need of a something whereby they are separate and distinct.* **This something is time.** We think about the separateness of events chiefly with regard to time, just as separate bodies carry [illegible] ith them the idea of [illegible]ace. It is [illegible] true that [illegible] ents may happen *tog*[illegible] *ime*, and they may *vary in nature*, but we are no[illegible] regarding them [illegible] this point of view. The words, no[illegible] w, then, before [illegible] er, can only be used

with reference to this idea of time, just as the words, here, there, up and down, for example, make space a necessary idea. **They are words which convey the meaning of separateness.** And time is the word which implies all this.

It is not difficult to understand that as soon as we gain this idea of time we realize **change.** We shall soon learn **that time is a necessary part of change,** and also that *all changes have to be compared with a given change* if we are to learn their true value.

117. In making a general review of the ground we have now covered, the first fact of importance to notice is, that we have dealt with the fixed and constant nature of **space** and **matter.** And it is here that our comparison of them with **time** must cease. All our measurements of space and matter have been based upon their freedom from change. Whereas in *time* we have an example of a new kind of measurement, that of a change itself. The meaning of time carries with it the notion of change. We have to treat *change as a quantity*, and *select some standard change whereby to measure it.* It is clear that the word change denotes the opposite of constancy. **But it is clear that change, just as much as those things which are constant, cannot be measured (or even perceived) unless there be something fixed to which reference may be made.** The same rule holds then for change as for that which is constant.

118. The beginning of knowledge as to time is that derived from our perception of an order in the occurrence of events. The idea of *order in time* follows from what has been previously said about *the separateness of events.* Beginners cannot be expected to go much more deeply into the subject than to recognize that the order and following on, or sequence of events, calls up the notion of time.

APPROXIMATE METHODS OF MEASURING TIME.

119. The practical measurement of time is now so easy, so familiar, and we are so dependent upon accurate knowledge of time in our daily life, that it is difficult to go back to the period when time could only be measured with difficulty, and **when accurate divisions of time were impossible.** All the business of modern life, the complexity of affairs, and even rapidity of communication, are made possible by the existence of *instruments* which readily indicate exact periods of time. It is true that the apparent motion of the sun, produced by the rotation of the earth, causes the prominent division of **day** from **night**; that the height to which the sun appears to rise in the sky determines the **seasons**, appearing highest in summer, and lowest in winter; and the appearance of the moon at its full marks the **months.** But much more than this is needed for civilization such as it is now.

It is not enough to know that day follows day, and month succeeds month, that years join themselves together in an unbroken sequence. How do we know that the appearance of the moon at its full, or the interval from summer to summer is regular? Do we know that the period of time between the highest point of the sun on successive days is the same? *What is there fixed?*

120. If we could *make sure in any way* that any of these **intervals,** to which the names of day, month, and

year have been given, were **fixed** and **unchanging,** we should have **something to start from, and a standard of time would exist.** We are forced to the conclusion that considerations of these periods of time alone can give us no help. The longer we ponder over the subject in our minds the more certain shall we feel our helplessness to be.

By a process of *counting, extending over several years,* we should finally come to the conclusion that there are **365 days** in each year, denoting by a day the whole interval, day and night, from mid-day to mid-day. But by continuing the process for many more years, we should find that this number is *not quite accurate.* Measuring from the day on which *the sun has apparently reached its highest point in the sky to the same day again,* as representing the interval of a **year,** we should find, in course of time, that there are **365¼ days** in each year. We should then, at any rate, possess a *Calendar.*

But even by such a laborious process we should only find out the *number of days in the year.* We should not know that this number of days *always represented the same interval of time,* for the **length of the day** might be an **inconstant quantity,** varying from year to year.

121. It is true we might *take for granted*—what is always assumed, in some degree, in our investigations of nature—**the general uniformity of nature.** But however strongly we may be urged to accept the truth of this principle in the case of changes which are under *our own control,* and which can be reproduced as *frequently* and *under such conditions* as may be desired, it is no inconsiderable assumption when applied to a change of such magnitude as that movement of the earth by which the years are marked.

But apart from the difficulties of this assumption, there is no need to point out that practical observations of the year are inconveniently long for a laboratory course. We may, however, confine our investigations to the day itself, and although there are serious practical difficulties, even in this case, yet we may consider a day to be a period more within the student's means of observation. Nevertheless, the *same difficulties* would again be encountered.

The uniformity of the length of day[1] **must needs be assumed before we can accept it as a standard.** We are, therefore, bound to admit that our personal investigations into the measurement of time must take another direction, if we are to learn experimentally how to ascertain equal intervals of time.

122. With regard to a possible subdivision of the day, we may direct attention to the fact, that a rough approximation of the "time of day" may be obtained by means of a **sun-dial,** an instrument consisting in its essential parts of a rod or bar, placed so as to throw its shadow on a flat surface which has been graduated. The position of the shadow will vary at different periods of the day. It is advisable to fix up such a rod in a favourable position, and to make a rough dial by comparison with a watch. But notice here that **we need a watch to make the dial.**

[1] The day as understood to mean the length of time during which the sun is visible must be kept distinct from the day measured from mid-day to mid-day. Our own sensations give us vaguely the information that the former varies according to the season.

If we can put out of our minds entirely all thoughts about the existence of such instruments as watches, we shall understand, from what follows, something about the reasoning involved in measuring time, and something of the methods of selecting a standard. *Assuming that watches do not exist,* we could make a very rough **timekeeper** out of the dial. We could mark out the area which the shadow sweeps out, and let the *portion of the area* swept out stand for a rough estimate of the *portion of the day* which has passed. The shadow would vary in size, and in position probably, at different periods of the year, although for several *successive* days we might depend upon the readings. Such a course would enable us to learn some of the difficulties of the subject.

123. A plan similar to this in method, and also in want of exactness, has been in use from the remotest antiquity. It is a method which serves for the night-time, and carries on from the day a means of measuring that mysterious motion of the earth, which causes the variation in the shadow of a sun-dial. We refer to the apparent **movements of constellations,** their rising and setting, or attaining their highest point in the sky. These various constellations being known, *their recognition at different parts of the sky* gives a rough estimate of the time.

In just the same way, the apparent course traced out by the sun between rising and setting, gives some indication of time, and it should be noted that **the cause of the sun's apparent motion is the same as that of the stars,** namely, the rotation or turning of the earth itself, the solid earth which appears so changeless.

PRACTICAL METHODS OF MEASURING TIME.

124. It will be readily admitted that for our purpose we require some change which can be proved to go on steadily or uniformly. It is clear that change gives the means of measuring time: that there is about change a something to be observed, which we call the *duration of the change*, or the *quantity of time*. We are aware of the *flow of time* by observing change. This is perhaps as much as need be said on this difficult subject. But we may repeat the statement, **that we need changes which take place at an even rate in order to compare intervals of time**, and these changes should be such as can be controlled and regulated by ourselves. The intervals of time which we call days are too long to form the subject of practical experiments, even if we take it for granted that they are *equal* intervals.

125. In ancient times an inaccurate measure of time was obtained by *sun-dials*, and instruments like our sand-glasses, but, instead of using the flow of sand, the time was measured by the flow of water. Such instruments were called *clepsydrae;* we now call them water-clocks. Either a **sand-glass** or a **water-clock** may be very useful in recording *equal portions of time.* It should be remembered that in respect of *all quantities*, whatever their kind, **the first process in measurement is to ascertain equal quantities.**

In the case of lengths and masses we come to an agreement in the first place, as to what is meant by equal quantities; and then we may afterwards proceed to *compare* one quantity with another by applying to them the same standard. A given quantity contains so many quantities equal to the standard, or, as we

say, contains the standard so many times. Another quantity contains the standard another number of times, and the *numbers* give an indication of the *relative value of the quantities.* We gather the numerical value of quantities when they have been measured by the *same standard.* **The same process must be followed with respect to time.** *The main difficulty is to select a standard.*

126. We may begin by selecting a standard which is quite **arbitrary,** that is, one of which the magnitude is not necessarily known except by those who select it, and by those to whom it is clearly described. We may say that our standard is the time occupied by the flow of 500 c.c. of water through a circular aperture of 1 cm. diameter in the bottom of a certain glass vessel, such as a beaker. This would be understood, and would form a fairly constant standard, but it would be difficult to produce such an aperture readily.

It would be better to select a glass tube through which the flow may take place, and make it of a known length and of known internal diameter. Such a tube should be obtained and fitted to a burette by means of an india-rubber tube. The time occupied by the fall of 20 c.c. of water from the burette may be compared with the time indicated by a clock. It will be found that the time as indicated by a clock, *which we may suppose to be correct*, will depend upon the actual level of the water. For example, the time

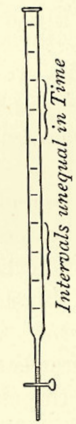


FIG. 89.—The lengths marked on the burette represent slightly different periods of time.

will be shorter from the mark 0 to 20 c.c., than it is from 60 c.c. to 80. It will form a useful exercise to take the time for each 20 c.c. of flow at the various levels. The flow may be stopped by the use of a clip. Several observations should be taken with the aid of a watch or clock.

127. Now it is apparent that if we know: (1) the change of level: and (2) the actual distance of the level of the water from the aperture: and (3) all the dimensions of the vessel, we have a *standard*, which may be carried about from one place to another or copied. **It forms a standard of time, by means of which other times may be measured and compared.**

We may much more safely rely upon our belief in the **uniformity of nature** in carrying out the observations described above. **All our observations would be valueless if similar causes did not always precede similar results.** Here we have precisely the *same conditions* in every respect. The bodies are alike, and the same change is taking place. All our observations of nature lead us to believe that *the same time is therefore occupied.* On the other hand, we are ignorant of the causes which may operate on the movements of the earth, and we have therefore less reason for assuming that they are uniform.

128. Instead of the flow of water through an aperture, we may make use of the flow of fine sand through an aperture, and the familiar *sand-glass* lends itself well to experimental purposes.[1]

[1] They have the advantage for class purposes of being cheap and handy. Their inferiority to burettes with water lies in the impossibility of varying their period.

In the same way we might take the burning of equal lengths of a *uniform candle* as giving equal periods of time. It is instructive to take a candle (such as those sold for testing luminosity, or *standard candles*) and compare the lengths of candle consumed (previously making marks at equal intervals) during successive periods of fifteen minutes each.

If we could make sure that the composition of the candle were absolutely uniform, and that there had been no irregularity in the burning, through draughts or other causes, we should have found an **instance of a uniform change which could be subdivided according to the lengths of the portions of the candle consumed.**

Much in the same way, we took a length of a column of water flowing out of a tube as indicating a certain interval of time. The latter change being so much more rapid, we should need to take a very broad column of water if we wished to compare these two methods of measuring time.

129. There is, however, a most valuable fact which can be learnt by means of our knowledge of the uniform nature of such changes as the flow of water and the even burning of a candle. **This fact is so valuable, that it is impossible to realize what would be the condition of our scientific knowledge, had it never been obtained.** I refer to the investigation of the swing of a *pendulum.*

We can take the time of swing, by comparing it with any other change which can be repeated under the same conditions, and we shall discover, that every observation points to the fact that **a well-constructed pendulum takes the same time to perform every one of its swings.**

Now the nature of this change is not easy to understand, and it will be very convenient to postpone for some time all questions as to the *causes* of the movement of a pendulum. We must be satisfied with the simple statement that the movement is produced by **the same cause as that which brings a body to fall to the earth**; in fact, a pendulum is formed of a body which *tries* to fall to the earth, but is prevented by the string or wire which is attached to it. With this explanation we must go on to make use of the *pendulum* as an *instrument for measuring time,* just as a steel or wooden scale may be used as an instrument for measurement of length, without any inquiry into its mode of construction.

(It may be explained to the student that the course here adopted is not that in which events actually succeeded one another in the history of science. Yet the gradual growth of correct methods of measuring time has arisen from much the same line of argument as that which has been expressed in the preceding pages.)

EXERCISES IN MEASUREMENT OF TIME.

1. Prepare a pendulum from a fairly heavy iron ball with a hook attached to it.[1] Tie to the ball a thin cord or wire of about 3 feet length, and make a loop at the end. Prepare another pendulum in the same way, but make it only half the length. Set them swinging together. Notice that the shorter one moves more rapidly than the longer. Find out how many times the one beats compared with the other.

We can affirm from this observation nothing more than that there is a *difference* in the duration of the two events being observed.

[1] If all the iron balls be cast so as to be approximately a pound each, they will be useful for subsequent experiments.

We have taken a convenient example *of events which can be observed at the same time.*

We can perceive when both objects are in corresponding positions, and we can perceive that one returns more rapidly than the other to that position.

Further than this, we can state that during a certain unknown period of time the one pendulum swings to and fro so many times, while the other pendulum swings to and fro another number of times.

But we have no ground for saying that the one pendulum moves so many times faster than the other, until we can find out that the separate movements of each take place in equal times.

2. Arrange a water-clock by means of a burette having a glass tube, with a fine opening, which is attached to its lower end by a piece of india-rubber tube. Consider that the time, during which the water falls from a given level (say that marked 100 c.c.) to another level (say that marked 50 c.c.), is always the same for successive observations. Refill the burette to the same 100 c.c. mark as required.

We can now observe that the pendulum beats the same number of times on every occasion that the water falls through the same difference of level. It may happen that the results given are not quite the same, but it will be found that the less the friction encountered as the pendulum swings, the better will the results agree.

In this observation it may be considered, that we have a means of obtaining equal periods of time, on the ground that exactly similar changes take place, namely, the same quantity of water falls from the burette under exactly the same circumstances. There is no reason to suppose that the period of time differs. *In fact, there is no other way of knowing what are equal periods of time.*

If we did not consider them as equal, we should have

no knowledge of time beyond that elementary knowledge of one event occurring before, after, or together with another.

We should not understand time as something capable of being measured, unless we could assert that the same change under the same conditions will always take place in the same time. *Hence we start measurement of time by deciding what are to be considered equal times.*

3. Count the number of beats during a smaller interval of time, that occupied by a smaller change of level of water in the burette, say from 100 c.c. to 90 c.c., and notice that there are the same number of beats when the pendulum moves through a large arc as when it moves only slightly. (That is, when the amplitude varies, the time occupied is the same.)[1]

Perform the same experiments as are given above, with a sand-glass substituted for a water-clock.

4. Observe that the pendulum which is one-half the length of the other swings four times to each swing of the other. Make separate observations, counting the swings of each during the same change of level in the water-clock, or while the sand-glass empties itself.

It may be inferred from these observations, that each beat of a given pendulum represents the same period of time, *and that we have in the pendulum a means of subdividing a given period of time into small equal portions.*

When we learn for a certainty, by using a pendulum which is more accurately made, that the time of swing is always the same, then we can feel sure that time can be measured, and that it is quite as much a quantity as length or mass.

[1] This statement is *only* approximately true. You will probably fail, nevertheless, to discover that it is not true, as the time is so nearly the same however wide the swing.

5. Make observations of the time of swing of your own pendulum compared with that of a standard pendulum, and draw up a table showing the relation of the lengths of pendulum to the time of swing.

A standard pendulum should be constructed of a massive "bob" suspended by rigid rod, which has fixed to it a "knife-edge" of steel so as to be able to stand on steel plates. This method causes very little friction, and gives a pendulum which will swing for a long time.

6. Assuming that equal periods of time are marked by the second-hand of a watch or clock at all parts of the day, construct a pendulum which will "beat seconds," that is, which will occupy a second in passing from one extreme point of its swing to the other.

Measure the exact length [illegible] dulum from its point of suspension to the centre of t[illegible]

7. Construct another pendu[illegible] actly the same length, but with a bob of different ma[illegible] easure from the centre of the mass in this case as before. Do this very carefully as the result is important. Ascertain if its period is the same.

8. Ascertain the time of beat of the given pendulum, and calculate its length from your result, knowing that the time is proportional to the *square of the length*, and that a second's pendulum is 99 cm. in length.

By the phrase "*proportional to the square*" we mean, that if the length is doubled, then the time is four times longer; if the length is one-third, then the time is one-ninth, and so on.

We may perceive that there is a connection between the standards of time and length, as well as between those of length and mass.

If we had no other means of finding out what was the quantity of matter denoted by a gramme, we could obtain it by finding out the quantity of pure water which would fill one cubic centimetre. (Practically this would be done, with less likelihood of error, by finding out first of all the quantity of matter contained in 1000 c.c. of pure water.)

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In a similar manner, we could re-determine the length of a metre, or rather of 99 cm., assuming it to be lost, if we were provided with a good time-keeper, by measuring the length of a pendulum beating seconds. Such a pendulum is 99 cm. long.

SUMMARY.

130. In the preceding sections a variety of phrases referring to time have been used, among them—*day*, *year*, *length of time*, *duration of time*, *interval of time*, and *period of time*. All these phrases originate in the fact that we regard Time as a quantity, which we may treat as other quantities have been treated. Just as we have measured and compared Lengths and Masses by means of standards, so we must proceed to measure and compare Time. **Length, Mass,** and **Time** are quantities which may be treated alike. In addition, they are the **three most important quantities** in existence. As we continue our investigations of nature, we shall find that all the measurements we may have to make hereafter are based upon measurements of some or all of these quantities. In other words, measurements of Length, Mass, and Time enter into all Physical Measurements.

But this is not all. Not only do they enter into other physical measurements, but there is a comprehensive system of measurement, whereby *all physical quantities* are *expressed in terms of* quantities of *length*, *mass*, and *time*.[1] Provided then we can find a suitable standard, time can be measured; a numerical value can be given to any quantity of time; and the treatment of

[1] The Centimetre-Gramme-Second System, or C.G.S. System, the so-called Absolute System of Measurement.

time differs in no way from that of other quantities. The selection of a standard, however, is a far more difficult matter than the selection of a standard of length or mass.

Length and Mass are constant objects, Time is something of a totally different kind. It does not remain constant while being measured. It is something which goes on or flows on without stop. We cannot *repeat* our observation of *the same time*, as we can of the same mass or length, for time goes on and eludes us. Hence we do not measure, in the strict meaning of the word, *time itself*, but rather measure *change* by means of a *standard change*. Still this is not the most accurate description of what is done.

More correctly speaking, we measure *one aspect of change*, which we call time, by means of the same aspect of another change which is selected as the standard. In measuring either length or mass we confine our observation to one property, neglecting all differences in kind or nature of the objects which are measured. So in measuring change we can pay attention solely to that aspect of change which is called time.

The year, month, and even the day are too long to be treated experimentally. Moreover we have no means of knowing that they are uniform quantities. The instruments which can be depended on for denoting equal intervals of time have the opposite defect of marking intervals which are too short. The water-clock, however, enables us to find out that the pendulum swings in regular periods of time. As a pendulum can be constructed to swing for a considerable fraction of the day, and it will swing for a number of days with the aid of machinery (as in a clock), we have means of comparing the length of day with the swing of the pendulum. We have also the means of comparing one day with another.

MEANING OF A SECOND.

131. Probably all who are working through this course have often been *told* that the earth rotates or turns round on its axis. This axis is an imaginary line. It will be understood that a top may be spinning in such a way that every portion of it circles round a line or axis. If this imaginary line be a *true line*, that is, one without breadth or thickness, we can perceive that all points in that line are at rest, and that this line may be pointing always in the same direction. This will happen when the top is *at rest as a whole.* But the top may spin round and move on at the same time. Now the earth is spinning round and moving on in the same way. *This statement ought not to be taken without due consideration.*

It is not easy to realize that the earth is spinning round, as is shown by the fact that until the days of Galileo many people much wiser than ourselves did not believe that it moved. **Either the earth or the sun and stars must move,** as we can see for ourselves; but it was not until very searching tests were made, that it became clear that the alternative which is so difficult to realize is the true one, and that our solid and mighty earth is in motion. We must accept the conclusions of the many skilful experiments, which have been carried out with great precision, and *try to understand* that the earth

is in motion. **It is the rotation of the earth, and not the movement of the sun and stars, which causes the apparent movement of the sun across the sky, the change of day and night, and the changes in the positions of the stars at night.**

132. We may readily perceive that the lengths of day and night are not uniform throughout the year. We can perceive this without the use of time-keepers, because we are directly conscious of day being short compared with the length of night in winter, while in summer the nights evidently are short. But any of our rough time-keepers would enable us to ascertain this fact with certainty.

We may succeed in *observing a day*, if we take up a suitable position with a piece of smoked glass (which enables us to look at the sun) and watch until the edge of the sun coincides in position with some fixed point on a chimney, flag-staff, or church tower, for example. Then if we choose to remain in the same position, and wait until the sun reaches the same point again, **we shall have observed the lapse of a day.**

Some means of *dividing* this day must be discovered, **for a day is too long a period for purposes of experiment.** Some such instrument as a *clock* must be used, **a clock being an instrument which registers by a skilful contrivance the beats of a pendulum, and describes on its dial how many beats have taken place during any given time.** Such an instrument would at once tell us that all days are not of the same length. Even two consecutive days vary to some extent, but the difference in the length of the days at different seasons is very distinct. The same result is obtained whether we use a clock, or any other instrument capable of giving us successive and equal intervals of time.

133. The question then arises, *does the earth turn round at different rates*, does it turn round more slowly in summer than in winter? If we had no further test to apply, we should have to admit that it does; for there can be no doubt that **the same change, produced by the same cause under the same circumstances, always occupies the same time**, and we have no doubt that the same change shown by the water-clock or pendulum denotes the same length of time. All kinds of tests being applied, we are driven to the same conclusion, namely, that days are not equal in length. Now it would be strange if all methods of measuring time were to agree in this respect and yet be misleading.

We are, therefore, forced to the conclusion, that the *length of day* is not a constant quantity and that it is misleading. *Still the variation in the length of day would seem to be due to other causes than an alteration in the rate at which the earth turns round.* It is difficult to understand *why* the earth should rotate more slowly, then increase its pace, and then again become slower, as it appears to do. It *might happen*, for aught that we know at present to the contrary. **And we should indeed be compelled to admit that the earth's rotation is not uniform, were it not for another observation which can be made, that is, the interval of time between the successive transits of a star past a fixed point, instead of the time occupied by successive transits of the sun.**

134. If the same observation as was made with the sun be made with a star, which is known not to change

its position in the sky as compared with other stars,[1] it will be found that *successive transits of that star occur at equal intervals of time.* **Hence compared with a fixed star the earth does rotate uniformly.** We ought perhaps to say that as far as we can judge the earth rotates uniformly. All kinds of tests, such as have been indicated, may be applied, but they all give the same result. If the earth do not rotate uniformly, then a great variety of observations must all lead to wrong conclusions; and instead of finding a general agreement among all our investigations, we should have a meaningless confusion.

135. We may now begin anew our explanation of the **meaning of a second. It is a certain fraction of the time occupied by the earth in turning completely on its axis**, and it is by observations of this time, by means of the transits of the same star across the cross-wires of a fixed telescope (or rather, of a transit instrument), that we obtain a check on our time-keepers. The mechanism of a time-keeper is adjusted, until it causes a given number of seconds to coincide with the period of two star-transits or **a sidereal day.**

It is decided beforehand that there are to be 86,164·1 of these seconds in a sidereal day. This seems an odd number to select; but it is due to the fact that ordinary people have to regulate their life, not by stars, but by the sun. And therefore they ascertain *the total length of all the days in a year* and then calculate the average length of a day, and obtain what is called the **mean solar day.** This is divided into 24 hours, 1,440 minutes, or 86,400 seconds.

[1] That is, a star as distinct from a planet.

136. Hence the *unit of time* which we call a **second** is based upon the average length of a solar day, and this is obtained from the total length of all the days in a year. But once having decided upon this quantity, any sidereal day affords a check of its value.

It is of course understood that throughout our investigation of the means of measuring time, we have used the word *day* with a special and limited meaning, namely, as the length of time in which the earth makes one complete rotation on its axis. But we may contrast with this meaning the loose and irregular use of the word in ordinary conversation. For example, we find it used with the following meanings:

Day stands for:

1. 24 hours or a mean solar day, *e.g.*, "The voyage occupied 5 days 14 hours."
2. The period between sunrise and sunset, a very irregular period, *e.g.*, "Which is the shortest day?" forms a frequent question in geography. Moreover in the neighbourhood of the poles it corresponds with 6 months.
3. The portion of a day spent in work, *e.g.*, "The eight-hours day."
4. Any indefinite period of time, *e.g.*, "Every dog has his day," etc., etc.

COMPARISON OF TWO TIME-KEEPERS BY MEANS OF A PENDULUM.

137. In performing this experiment it must be *taken for granted that the time of vibration of the movable pendulum does not vary*, otherwise the fundamental principle of measurement, that the standard which is used must not change, will be violated.

We are about to use the time of vibration of a pendulum as a fixed quantity of time, and measure a long interval shown by the standard clock, by means of finding how many times this small selected interval is contained in the larger interval.

The time of vibration of the pendulum is our *unit*, and the number of vibrations which it executes in the long interval gives the numerical value of this interval in terms of our unit.

Now, just as in the measurement of length, we can reproduce a given length which has been expressed as so many times a standard length, by setting out that number of times the standard length along a straight line; so we can *reproduce a given interval of time*, which has been measured as so many times a certain standard interval, by allowing that number of times the standard interval to elapse.

We see then, that if we wish to compare a timepiece with a standard clock, we must take a known interval, say 3 minutes on the standard clock, and find how many vibrations the movable pendulum makes in that interval. Suppose it makes n vibrations. We then remove the pendulum from the standard clock; swing it near the timepiece; and find the measure of the interval on the timepiece during which the movable pendulum makes n vibrations.

As an experiment, the time of vibration of the metronome may also be compared in the same way with the clock by means of the pendulum.

The pendulum used for this purpose consists of a flat steel bar. The thickness may be about one-quarter the breadth. Through a brass block attached to one end of the bar a piece of steel wire passes horizontally and forms the axis of vibration. The support consists of a plain piece of wood with a rectangular hole in it. This can easily be clamped to the edge of the bench. At the other end of the bar is a hook, to which a weight can be attached, to steady the motion and make the pendulum swing for a longer time.[1]

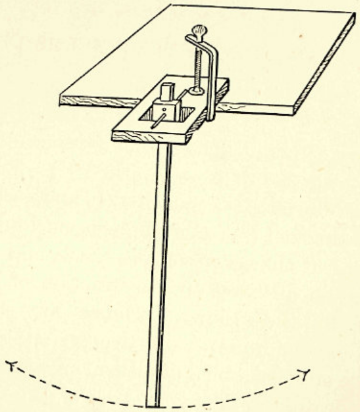


FIG. 90.—A useful form of pendulum and stand for experimental purposes. It may be clamped to a shelf or bench.

Two observers will be necessary. In the first part of the experiment one of them notes the interval on the standard clock, while the other counts the vibrations of the experimental pendulum.

Enter thus:

Interval on Clock, - - - -	180 seconds.
Vibrations of Pendulum, - -	272.

The next step is for one observer to count 272 beats of the pendulum, while the other counts the number of beats made by the metronome.

Enter thus:

Number of beats of metronome in 272 beats of pendulum = 211.

[1] The pattern of the pendulum adopted need not resemble the one here described, but steel bar is readily obtained at the ironmonger's, and the lengths now required will serve for various experiments afterwards.

272 Beats of pendulum = 180 seconds.

∴ 211 Beats of metronome = 180 seconds.

Number of beats of metronome in 180 seconds found by direct experiment = (   ).

138. EXPLANATION OF SOME TERMS USED.

NOTE.—*The following statements are of little value unless they are accompanied by detailed demonstration with the aid of a globe and models.*

DIVISION OF THE CIRCLE.—The circumference of every circle is divided into 360 degrees, marked (°). Each degree is divided into 60 minutes, marked (′), and each minute into 60 seconds, marked (″). An angle is enclosed by any two lines meeting at the centre of the circle, and its magnitude is measured in all cases by the fraction of circumference subtended by those lines. Hence any angle is fully described by the number of degrees it contains. The convenience of this method of measuring angles is especially marked in dealing with the rotation of the earth. In one complete rotation a given point on the surface moves through a circle, in half a turn it moves through 180°, and so on. When the earth rotates on its axis all points on its surface except the poles turn through 360 degrees in a sidereal day. The angular distance through which the earth moves is the same as the angular distance through which a given fixed star appears to move. This is measured by any instrument indicating angles, the only difference being that the angle appears to be traced out in the contrary direction. Hence it is that movements of the earth are always ascertained by observing the apparent motion of distant stars.

SEXTANT.—The sextant is an instrument used for measuring angular distances between stars, or for measuring the altitude (that is the angle from the horizon) of the sun or other bodies. The instrument consists of the sixth of a circle finely graduated, carrying a movable arm with a mirror, a telescope and a fixed mirror.

TRANSIT INSTRUMENT.—This consists of a good telescope fitted to a vertical graduated circle, so that the angle through which the axis of the telescope is rotated at the centre of the circle may be measured. (By this means angular distances in a given plane may also be ascertained.)

Pole Star.—As the earth rotates and a person on its surface is carried round with it, the fixed stars, which are at an enormously greater distance than any distances which exist on the earth, appear to move at the same rate in an opposite direction. As the earth moves round from west to east, the stars appear to move in a body from east to west. But as the North Pole is approached, the observer would move through smaller and smaller circles. Many stars too would seem to move through a smaller circle, of which the centre is the Pole Star. At the North Pole itself the Pole Star would be always directly overhead, while other stars would appear to sweep round parallel with the horizon (the line where the sky and earth seem to meet). In our own latitudes the Pole Star has an altitude (or angular distance above the horizon) of about 50°. Hence there are many stars visible which circle round it and never set; but there are about the same number which both rise and set. There will be of course many stars which are never visible unless we go further south.

Second.—A certain fraction of the time occupied by the rotation of the earth on its axis.

Day.—The time occupied in the rotation of the earth, as measured by the sun, is variable through the year. The mean time calculated from the year is called the *mean solar day*, and contains 86,400 seconds. The rotation as measured by a star is called a *sidereal day*, and contains 86,164·1 of the same seconds (*mean solar seconds*, or seconds obtained by subdividing the mean solar day by 86,400).

Year.—The time occupied by the earth in completing its journey round the sun. The path taken is called the *orbit* of the earth. The highest angular distance attained by the sun being noted, the completion of the orbit is marked by the sun *again attaining that height*. The plane containing this orbit together with sun is called the *Ecliptic*. The angle made by the axis of the earth with this plane is always about $66\frac{1}{2}$°.

Exercise.—Knowing that the Earth's axis is so inclined to the Ecliptic, and remembering that the earth has two motions, a daily rotation and a yearly revolution, how do you explain that at the pole, and some distance around it, the sun is visible for 6 months and invisible for the rest of the year?

Pendulum.—Any body, so placed that it can freely swing to and fro under the action of the earth, can serve as a pendulum.

Clock or Watch.—An instrument which serves to mark equal intervals of time, and also to record the total time between any two events. By means of a mechanism, consisting in the main of cog-wheels connected so that a movement may be passed on from one to the other, the motion caused by the fall of a weight or the uncoiling of a spring is recorded. This motion is kept constant by the vibration of a pendulum or wheel which has been previously regulated to vibrate in seconds.

FURTHER EXERCISES IN MEASUREMENT OF TIME.

1. What is the time when the hour-hand of a watch stands at the third minute-mark past the hour-mark 12?

2. At what time between three and four o'clock will the two hands of a watch be in the same straight line?

3. The minute-hand of a watch is $\frac{3}{4}$ inch long, and the hour-hand is $\frac{1}{4}$ inch long; find the distance traversed in a day by the extremity of each.

4. Through what angle does the hour-hand of a watch move in $3\frac{1}{2}$ hours?

5. If the earth turns through 15° in an hour, that is, if it performs $\frac{1}{24}$th of its complete rotation in 1 hour, what will be the time at Philadelphia which is 75° west longitude, and at Calcutta which is 88° 25′ east longitude, when it is noon (12 o'clock) at Greenwich? To answer this question, it is only necessary to remember that noon is the moment when the sun passes the *meridian*, that is, the vertical plane passing through north and south.

6. How much earlier does the same star cross the meridi each night?

Remember that the earth has two motions, in consequen of which there is a daily apparent motion of the stars, and yearly apparent motion of them which is added to the dail motion. If the earth were not rotating at all, the yearl revolution of the earth would cause the same apparent change in the sky at night as now occurs in 24 hours.

7. What is necessary for finding out the position of a ship at sea, assuming the necessary instruments and almanacs on board? To answer this question, remember that we to find out longitude and latitude. Longitude is learnt finding out the highest position of the sun, and the time indicated by a good chronometer. How is this done? Ho will the position of the Pole Star, or the sun at its highest poin vary with the latitude?

8. Suspend a small magnet by a fine fibre of silk, and show that it makes all its vibrations in equal times (that is, its vibrations are *isochronous*). To do so count the number of vibrations during successive minutes, or longer periods. Show that the same statements hold, for a stronger magnet,[1] but that the vibrations take a shorter time.

9. Fix to the middle of a large mass, a cube or sphere of iron or other metal weighing from 20 to 60 pounds, a steel wire, which is thick enough to support it with safety, but not too thick. The wire should be *rigidly* fixed to the mass, and should be firmly and tightly supported at a height to allow at least 6 feet of wire to be tightly stretched by the mass. Fix a small mark or pointer on the mass, twist it round about twice, and leave the wire to twist and untwist itself under the action of mass. Prove that the movement is isochronous; and that the apparatus is capable of acting as a time-keeper. It is essential that the fastening at each end should be firm enough to prevent the wire turning at the top, while at the bottom it must move through the same angle as the mass itself.

[1] The stronger magnet should not be bigger, or it *may* swing more slowly; in fact, a change, which will be observed in the next question, may counteract the change due to magnetism.

CHAPTER VIII.

MODES OF RECORDING OBSERVATIONS.

USE OF NAMES AND LANGUAGE.

139. It is one thing to observe. To convey to others what we have ourselves observed is another matter. There is often as much need of care in the description of what has been done, as in the method of doing it. Yet the true study of science engages us in learning about *nature,* not about *names.*

We are now occupied in *observing* the objects around us. Later on we shall have to use our reasoning powers to make *inferences* or draw conclusions from what we observe. In describing our thoughts we shall have to use language, and we have already had to describe what we have seen. But the main intention has been to observe, the act of measurement being the chief part of this work. In measuring, we have merely observed more closely and accurately. The use of names has been secondary to the use of our senses.

At the same time it is important that when names are used, they should be rightly used; that the same name should be the sign of the same object. When the word *sphere* is used, it is essential that it should convey **the same meaning to all.** If it serve this purpose it will be *in consequence of all our thoughts about the sphere being alike.* To know the nature and properties of a sphere we must have it before us, we must see it, and find out by observation what it is like.

140. A name, as *sphere,* may stand for an individual object which we are observing; but we pass from the **individual** to a **class** in learning that there are a large number of such bodies. We may learn facts which are true of that individual sphere, such as its exact diameter or its colour, but in the main we have learnt facts that are true of all kinds of spheres. Hence we can make *a general statement or proposition about spheres,* for example, that every point on the surface of a sphere is equally distant from its centre. This is an important thought, which the name sphere ought to raise in the minds of all who use that name. There are other thoughts, too, which the name should raise, but if we combine the thought just mentioned with the thought that a sphere is a solid figure, we know all the *essentials* of a sphere and **we have defined it.** But it may, perhaps, be as well to know how to *describe* objects for ordinary purposes before we attempt to define them.

141. There is nothing more important at the beginning of the study of science, than to be able to write out in a simple but exact manner a description of what has been observed. And there is nothing that is rarer than excellence in this respect.

Constant exercise is needed to gain any facility in description.[1]

Now a **description** of an object may be satisfactory in some respects, and unsatisfactory in others. It may be sufficient to denote what is meant on some particular occasion, as, when we say a man is a two-legged animal, we state one difference between a man and a caterpillar. Or, again, in saying that a measurement was made with a metre scale, all the *necessary* description may have been given, but neither descriptions would be *complete.* Whether a description is sufficient or exact enough depends upon the purpose in view.

All that which is an essential or a necessary part of any occurrence or fact must be expressed in words if a description is to be good. And the first and chief aim of all attempts at description should be to make sure in your mind *what it is that you have observed.* Then you may use fitting language to express it. This is no easy task; and one of the chief aims of your work will be to carry on this most important training, namely, **the separation of fact from fact, and the direction of the senses to one property while others are neglected.**

142. Correct observation, then, is the first step, and adequate description must follow it. When many objects or events have been observed *to be alike in*

[1] It is advisable to give examples of descriptions, and then set questions on the same subject. For instance: a hat is a covering for the head. A box is something made of wood or other material joined together so that it may serve the purpose of holding bodies. A book is a collection of sheets of paper with printing thereon. Now, describe a pen, candle, table, football, and collar.

some respect, we are in a position to make a *generalization.* We begin to form a **class,** and a certain de[illegible] **statement or proposition may be made about all the members of the class.**

For example, we notice that in all cases where we have measured the mass of 1 c.c. of pure water, we have found it equal to 1 gram. This is the simplest form of generalization, and yet there must be an accumulation of observations to enable us to make the general statement that any c.c. of pure water weighs 1 gram. It is, of course, easy to use arguments, such as *the recognized uniformity of nature;* to assert that it is in the **ordinary course of nature for matter to be uniformly distributed in the same body;** but these statements are based upon the experience of innumerable instances of this and like cases.

143. A generalization of a different class is made when we discover all circles to have the circumference 3·1416 times the diameter. This is a generalization which is recognized to be *a necessary truth for all circles,* as in all circles there must be the same relation between the diameter and the circumference. Or again, when we say that every point on the circumference is equidistant from the centre of a circle, we express a general truth or generalization which also serves as a definition of all circles.

A definition is a description so complete that an object may be recognized readily from it. A definition does not necessarily describe the most prominent properties, but **it must give those which are essential.**

We may notice that names do more than merely *point out or denote.* Terms like square or circle bring up in the mind the thoughts of certain properties

which belong to these objects in general. One or other object may be **denoted,** but the *properties of the object* are **connoted** at the same time.

MODES OF RECORDING OBSERVATIONS BY USE OF LINES.

144. We have learnt already that observations may be recorded with accuracy, by the use of terms and language which have been selected according to certain rules of exactness. It must be granted, for example, that **a given term shall always stand for the same object or quality;** that the word *cube,* or the word *size,* shall always bear the same meaning for those who use it.

When a given term is used, it ought to *convey to the minds of others such thoughts* about that thing, of which it is the sign, *as we ourselves have in our own minds.* **And the thoughts of each one, when tested by the same questions, should produce always the same answers.**

But the use of *language,* unless quantities alone are referred to, often leads to *inaccuracy;* and long before we can trust ourselves to learn about nature *from description,* we should occupy ourselves with such observations as can easily lend themselves to *measurement.* It is for this reason chiefly, that the study of **natural science should always commence with accurate methods of measurements and accurate methods of record.**

145. All the quantities, which have been measured so far, have been such as could be represented by *a number* and the *name of a unit;* and no quantity can have its value, or numerical value as it is called, represented in words except by a phrase which has two parts or components, a number and a unit. For example, in describing a length we say *three feet, one metre, two millimetres.*

It is true we might describe a variety of lengths by merely giving a series of numbers, to which the name of a unit is supposed to be attached. We say the dimensions of a room are 24 by 20, meaning 24 by 20 *feet*. But when *various units are in use* it is important to state which one is meant. **If a universal unit of length were in use, numbers alone would describe any given length adequately.**

146. Now if we fix upon a given length, and draw a line which shall have as many *millimetres* in length as a given object has *centimetres* in length, we shall have **a line standing as a record** of the length of this object. The line would in fact have one-tenth the length of the object measured. We then say it is *a record drawn on the scale of one-tenth*, or, for shortness, on a $\frac{1}{10}$ scale. Keeping to the same scale, or to any other which may prove convenient, we may represent any number of lengths by lines drawn to correspond with them. For this reason we can *recall* from the recorded lines the **lengths for which they stand.** All that is needed is to measure the lines, and then *change the unit.* A line of 15 mm. on a $\frac{1}{10}$ scale will express a length of 15 cm.

147. But it is not only lengths which can be represented in this way. A line of 1 mm. may be used as a record of 1 gram, and then *any number of grams* may be represented by *lines*, drawn to correspond with them by the number of millimetres it contains. A line 12·5 mm. in length would denote 12·5 grams.

In just the same manner *time* may be recorded by lines. It must be remembered then that the **three primary quantities, Length, Mass, and Time, may be represented by means of straight lines drawn on paper, the length of line denoting the numerical value of the quantity according to a scale determined on beforehand.**

In addition, we may mark off upon lines lengths which shall correspond with *any kind* of quantity besides those mentioned; and a mere inspection of the lines will at once convey to the mind an impression of the relative values of these quantities. Perhaps it is the greatest advantage of this mode of representation **that it does immediately convey to the mind in a simple and direct manner what is intended to be expressed.** Such a record is said to be *graphic.*

148. It is not difficult to perceive that if our record is a true one, the lines drawn *ought to correspond exactly* with the values recorded. This implies that the accuracy of the drawing is quite equal to that of the measurement. We should be able to measure off lengths of lines at least as accurately as we have been able to measure the quantities to be expressed. Now, we may frequently find a quantity which cannot be stated precisely in terms of a given unit. For example, a distance to be measured may not be any exact number of centimetres, or even millimetres, small as a millimetre is for a standard. It is true we might subdivide the millimetre, and express the extra length as a fraction; but on employing a microscope for greater accuracy, we might then find that it did not quite correspond with the supposed fraction. Similarly, we may employ in any measurement more and more accurate instruments or means, and come to the same conclusion, that we can do nothing more than measure approximately. **At the same time, such measurements may be said to be accurate, if they serve their purpose accurately.** The same statement may be made, whatever may be the quantity measured.

More and more accurate means reveal greater and greater deficiencies in the accuracy of the measurement, whether it be of length, time, mass, or any other quantity. But on this ground there is no fault to be found with the graphic method, for a line can be measured with as great accuracy as any quantity.

EXERCISES IN REPRESENTATION OF QUANTITIES BY STRAIGHT LINES.

1. Draw lines to represent 1·5 and ·35 metres to $\frac{1}{10}$ scale (1 mm. represents 1 cm., or 1 cm. represents 1 decimetre).

2. Taking a length of 1 cm. to stand for a gram, draw lines to represent 5, 12, 6·5, and ·9 grams.

3. Measure the length and breadth of the room in feet, and draw lines to correspond, representing every foot by $\frac{1}{8}$ of an inch. The scale is called for shortness $\frac{1}{8}$ scale, and is the one most frequently used in architects' plans.

Proceed from these two lines to represent the room in plan, drawing on paper an area which represents that of the room.

Prove that your area contains as many squares, each $\frac{1}{8}$ of an inch in the side, as there are square feet in the room.

4. Measure the room again in metres, or calculate the dimensions in metres from the value in feet, and draw a plan to the scale of 1 cm. for 1 metre (or 1 mm. for 1 decimetre).

How will your two areas be related to one another?

5. Measure all the dimensions of a box in centimetres, and draw them to $\frac{1}{100}$ scale. Draw a plan and elevation of the box, that is, one horizontal and one vertical surface, and show that they form a record of all the external dimensions of the box.

6. A plan and elevation of a box are drawn to $\frac{1}{8}$ scale; calculate the total volume from the plans given you.

7. Draw a ground plan of the room, showing the position of the tables and benches in it. Use $\frac{1}{100}$ scale.

8. Taking 1 mm. to stand for a minute, draw straight lines to represent 5 minutes and 150 seconds.

9. Taking 1 mm. to stand for an angle of 1°, draw straight lines to represent angles of 15°, 35°, and $\frac{1}{3}$ of a right angle.

10. Representing by 1 mm. a value of 1 gram, 1 sq. cm., and 1 c.c. respectively, draw lines to represent the following quantities: 13 grams, 4 c.c., the volume of 1 cubic decimetre, the area of a circle of 3 cm. radius, the area of a circle of 6 cm. radius, the area of a square foot, and the volume of $\frac{1}{2}$ pint.

11. Make a dimensional drawing of the stand given to you, showing a plan and section, of any suitable scale. Satisfy yourself that all its dimensions are represented by your plan and section.

EXERCISES IN THE USE OF SQUARED PAPER.

The use of *squared paper* for recording the results of observations is so general, and it is a method so valuable, that exercises with squared paper may begin at once. For beginners, copy-books may be obtained with their pages ruled into squares of about $\frac{1}{4}$ inch square.

For more advanced work, paper with smaller sections (millimetre, or tenth-inch squares) will be needed. The counting of the squares in this case is aided by means of coloured lines, which show squares of 25 times and also 100 times the area of the small squares.

We may notice that the use of squared paper not only enables us to make *rapid records of areas*, in itself an important point, but it enables *calculations with regard to areas* to be rapidly carried out.

After exercises on this use of squared paper, we shall make ourselves familiar with the much more valuable services which it can lend, in enabling us to make **records of two quantities by a single operation.**

1. Notice the relation between linear distance and number of squares, in a larger square drawn upon the paper. Without measuring by any other standard, take as unit of length the side of a small square, and notice that squares of 4, 6, and 8 units of length in the side contain 16, 36, and 64 small squares.

2. Draw upon the squared paper any right-angled triangle with two sides coinciding with two lines of the paper, and note that it is always one-half of a rectangle of the same base and height.

You will perceive that one side of the triangle acts as a diagonal to the rectangle, and it cuts the squares in such manner that it leaves segments with *counterparts on each side the line.*

3. Draw a triangle, not a right-angled one, and show that it is one-half a parallelogram of the same base and height.

Then show that the parallelogram is equal to a rectangle on the same base, and of the same height.

Hence the area of any triangle may always be ascertained as half that of a rectangle of the same base and height.

The same facts may be proved geometrically, or by cutting paper to the required shapes.

4. Draw circles on squared paper with radii in the relation of 1, 2, and 3, and show by means of counting the squares that their areas are related as 1, 4, and 9, in just the same way as squares are related to their linear dimensions.

5. Draw several rectangles of the *same area*, varying the relation of the sides to one another, but maintaining their product unaltered: for example, 8×6, 12×4, 16×3, 24×2, 48×1, and notice that they contain the same number of squares.

6. Trace on the squared paper any irregular surface, such as that of a fragment of paper, and calculate the area of your tracing by measuring that of a single square, and counting the number of squares. A number of pieces of paper cut together, so as to have the same area, should be given to the class.

7. Draw a plan of the room, taking the side of one of the squares of paper to represent one foot. Show in your plan the right position of the tables in the room. Calculate from your plan the area of the floor which is unoccupied by tables.

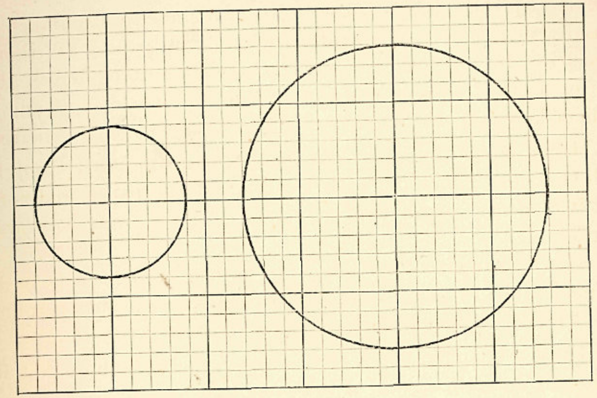


FIG. 91.—Two circles with radii as 1 to 2 may be seen to possess areas as 1 to 4.

8. Draw a hexagon (a six-sided figure with all its angles 120°) on the line made by the sides of three adjacent squares, and find out the number of squares it contains. Show that its area is the same as the sum of the areas of the six equilateral triangles, which are made by drawing lines from its centre to each of its angles.

Also ascertain its area by dividing it into three parallelograms.

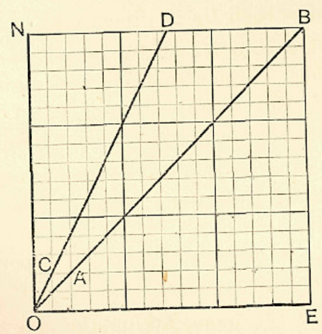


FIG. 92.—The line AB is equidistant from OE and ON at every point. The line CD is at every point twice as far from OE as from ON.

9. Draw at right angles two lines to coincide with those of the paper. Then trace out a line which shall be always equidistant from each of the given lines.

10. Draw lines which shall be throughout their length at a distance of 2 to 1, and also at 3 to 1, from the two lines respectively.

11. Find a point which is distant from the two lines respectively in the relation of 13 to 11. Also a point in the relation 22 to 19.

(In each of the cases place the points as near as possible to the angle, consistently with accurate measurement.)

Then join either of the points with the angle by a straight line, and make it clear to yourself, that *every point* on that straight line should *maintain an unchanged relation*, as regards distance, from the two given lines at right angles.

12. Divide a line, equal in length to the sides of 20 squares, into two parts, which shall be in the same relation as the numbers

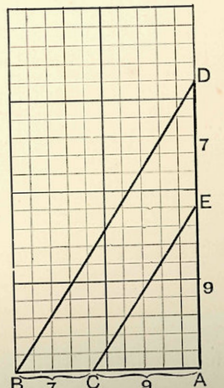


FIG. 93.—The line AB is divided at C into two parts, which are related as the numbers 9 and 7.

9 and 7. Count off distances equal to 9 and 7 sides of squares successively along a line at right angles to the first. Join the extremity of the first line to the last point marked on the other, and then draw a parallel line from the first marked point.

FURTHER EXERCISES IN THE USE OF SQUARED PAPER.

Two Quantities of the same kind represented by a Curve.

1. Use squared paper, draw two lines at right angles to each other, and find a point which shall represent, on $\frac{1}{100}$ scale, the position of an object 2·5 metres from one wall of the room and 3·75 metres from the other.

2. Draw a line on squared paper which shall show the relation between the length and breadth of a rectangle, which alters in shape but maintains the same area.

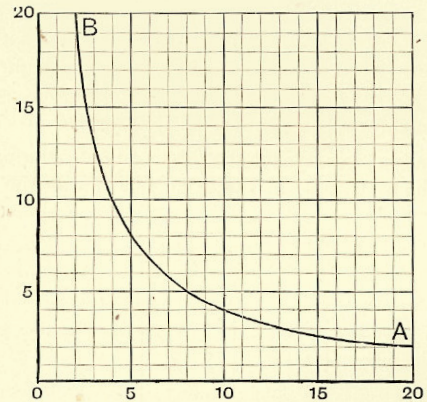


FIG. 94.—AB is a line all the points of which describe, by means of the distances from the two co-ordinates, a rectangle of the same area.

If we count units of length along one co-ordinate, and units of breadth along the other, we can indicate a succession of rectangles of the same area, as may be gathered from an inspection of the line AB in Fig 94. This line may change its position on the diagram, but its shape will not alter.

3. Draw, in the quadrant of a circle of some 8 cm. radius, lines to mark off every angle 5°, 10°, 15°, 20°, etc., up to 90°. Then draw chords to these angles.

Draw two lines at right angles on squared paper, along one measure angles, and along the other measure the chords.

Draw the line which will show the relation between the magnitude of an angle and its chord.

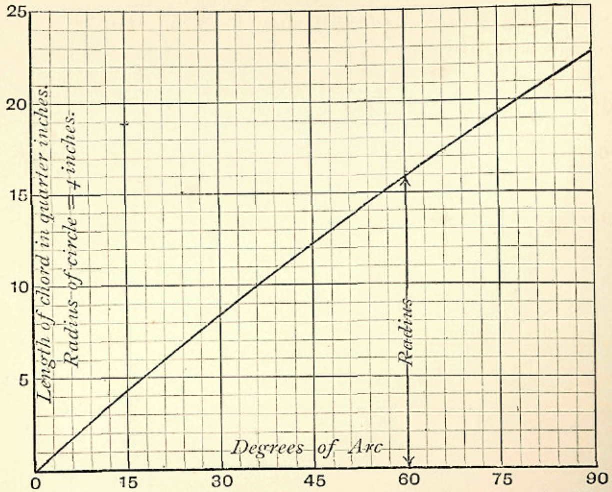


FIG. 95.—*AB* is a line showing the relation between the magnitude of an angle and its chord.

4. Carry out the same observation, but instead of drawing the chords, draw perpendiculars from the extremities of the tracing line. Make a record of your results in the same way as in the preceding exercise.

5. Making use of the plan recording the relation existing between an angle and its chord, find out from your curve the angles which, having sides of the same length as those recorded, would have chords of 8, 9, and 10 units of length respectively.

6. Also, find from your other curve the angles, of which the subtending perpendiculars have ·6, ·7, 1, 4, and 7 units of length, the radius of the circle being 10 units.

7. Measure the angle, made by two points on the wall with a third on a level with one of them (a theodolite may be used), and then measure the distance of the third point (the axis on which the telescope turns) from the wall. Then make use of the curve constructed in (4) to find the value of the perpendicular (the distance between the points supposed to be unknown).

8. Draw a curve which will show how the area of a circle increases with its radius, representing the fact that it increases as the square of its radius, taking 1, 2, 3, 4, 5, etc., units as radii.

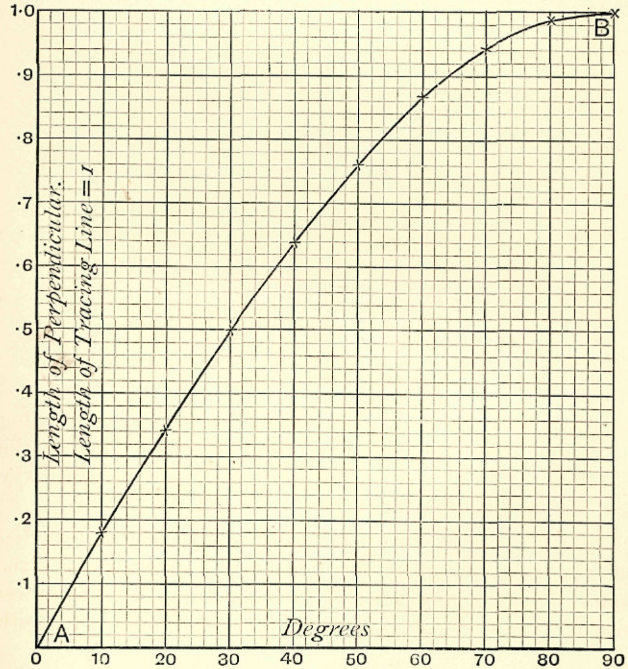


FIG. 96.—*AB* is a line showing the relation between the magnitude of an angle and that of the perpendicular dropped from the extremity of the line tracing out the angle.

9. Draw another curve representing the fact that the volume of a sphere varies as the cube of its radius.

NOTE.—It may be pointed out that the word *curve* is used in speaking of any line, whether straight or curved, used in diagrams of this kind as a record of the relation between two quantities.

SUMMARY.

149. In the preceding exercise we made observations which can be looked upon as the measurement of two quantities of the same kind, and we have been able to make a record of the results in such a way that **the same sign marks out the two quantities.**

The two straight lines which have been drawn at right angles to one another may be called the *co-ordinates.* By means of these co-ordinates, *two quantities may be represented at one and the same time.*

It will be admitted that there is no possible objection to a given length of line standing for another distance. We may represent a mile by a millimetre if necessary. Moreover it is clear that a convenient record of *other quantities,* time and mass for example, may be made, by means of straight lines of lengths which have been agreed upon beforehand.

But by referring to two straight lines at right angles, we can represent at the same time two linear distances. For we can draw a line at any distance from one of these lines so that all points on it are equally distant from it (*i.e.,* parallel to it), and we can select such a point on this line that it is *at any required distance from the other line.*

Hence such a point gives two lengths, and these two lengths may be the records of any two quantities we need to express.

It is clear that there would be no gain in making use of this method of expression unless there were some connection between the two quantities. **It is for quantities which are connected together that the method is used.**

There is no difficulty in quoting examples of such cases. The area of a circle grows larger as its radius increases. The cost of goods increases at the same time as the quantity. The size of a growing boy increases with years. The tightness or pressure of air within a football depends upon the quantity of air forced in. These are a few examples of the numerous quantities which are found to be connected together, and their connection is most directly and conveniently expressed by means of curves.

It will be noticed that **we simply make use of the two dimensions of a plane surface.** Our records are, as a rule, expressed on paper, and therefore we take *such means as a plane surface presents.* That is, we measure so many steps in one direction, and so many in a direction at right angles, one direction being lengthways, the other broadways. *We have carried on the same operations as are needed in describing the position of a point in a plane.*

In the exercise given above, quantities of *the same kind* were given, but we shall soon proceed to cases where the quantities are *not of the same kind.*

150. EXPLANATION OF SOME IMPORTANT TERMS.

OBSERVATION.—Whenever our senses are affected or acted on in any way we are said to observe. To see, to hear, to touch, to smell, or to taste, is to observe.[1] Whatever we observe is outside ourselves, and by the act of observation we are placed in connection with the world which exists apart from ourselves. We may divide what we can perceive or notice by means of the senses into two classes, namely, *objects* and *events,* or, *objects as they are* and *objects as undergoing changes of various kinds.*

EXPERIMENT.—Changes may be either such as occur independently of ourselves, or they may be caused or set in motion

[1] Using the word *observe* in the simple and direct sense which is necessary in this connection, and is employed throughout this book.

by our own acts. Those changes which go on in nature apart from ourselves may, or may not, be observed by us. But it sometimes happens that we can alter or control such changes, and if we do so for the purpose of adding to our knowledge, we perform *an experiment*. In addition to this we may bring together, order, or arrange objects so that they may undergo change. If we do this with a view to observe what takes place, we make *an experiment*.

LANGUAGE.—The impressions made by external objects and events upon one person, are made known to others by means of words. The same word ought, on all occasions, to stand as a sign of the same object or event, so that, when it is spoken or written, it may have on the mind of the person hearing or reading it, an effect similar to that existing in the mind of the person using that word. A word may also serve as a sign of a *thought* about an object or change. Hence it is that language is a means of communicating or recording *observations*, and *thoughts* based upon observations.

THOUGHT.—Our minds are so constituted that impressions received from the external world linger for varying periods of time. We are, therefore, able to call up or recollect past impressions as well as to receive fresh ones. A new observation may therefore be made at the same time as we recall one or more past impressions, which are similar in kind or in any way connected with it. In such case we are in a condition to *think about* the object or event which is observed. We must be careful, on all such occasions, to keep the impression, which is received during an act of observation, quite separate and distinct from the thoughts which are called up in connection with it. In other words, it is important to distinguish what is observed from what is *inferred*.

CHAPTER IX.

MEASUREMENT AND RECORD OF TWO CONNECTED QUANTITIES.

MEANING OF DENSITY.

151. We have already learnt that in **the same kind of matter the mass is always directly proportional to the volume,** that is, mass and volume increase together *at the same rate*. There are, for example, in 10 c.c. of water 10 times the mass existing in 1 c.c. This statement is in agreement with all our experience. We have no reason for supposing that there is anything but uniformity in the composition of water, and all experiments show that it is uniform. If we found any deviation from the statement now quoted, we should *suspect the presence of some kind of matter which is not water*. The same argument holds for other kinds of matter.[1] All kinds of tests confirm this general truth of nature, that **under the same conditions the mass varies as the volume.**

In the following diagram drawn on squared paper,

[1] The argument assumes that difference of physical conditions no more exists than difference of chemical composition. The result of change of conditions must be learnt from the study of physics.

that law is expressed by the line AB. Units of length along one co-ordinate stand for units of mass, and units of length along the other represent units of volume. The line AB is formed of a succession of points, each one of which marks the magnitude of both mass and volume at the same time: mass by the distance from the vertical line: and volume by the distance from the horizontal one. *The units of length* measured along each co-ordinate are *equal*, that is, the side of each square represents a unit of mass or volume, and in consequence of this,

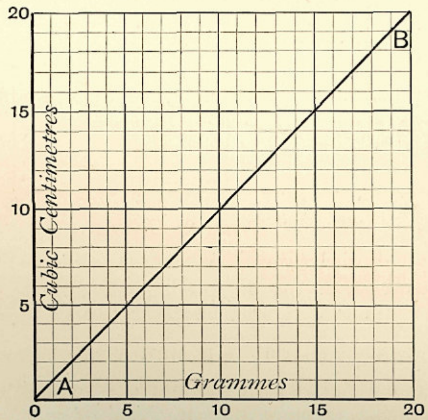


FIG. 97.—Diagram showing by the line AB the relation between the mass and volume of water. The mass is measured along the horizontal co-ordinate, *i.e.*, from left to right, while the volume is measured by the vertical distance.

the line AB is *equally distant* at all points *from either co-ordinate*. An inspection will make this clear. Now this diagram exactly represents the relations between the numerical values of **mass and volume in the case of water,** for the unit of volume, 1 c.c., does contain the unit of mass or 1 gram.

152. But we will now consider how we may represent by a diagram the relation between the numerical values of mass and volume, in the case of a kind of matter, such as *brass*, which contains about *8 times the unit of mass in the unit of volume.* Taking another piece of squared paper and drawing co-ordinates, we find a point which represents 8 divisions along the line of mass, and 1 division along the line of volume. Another point representing 16 units of mass and 2 units of volume is next found, and so on. Then

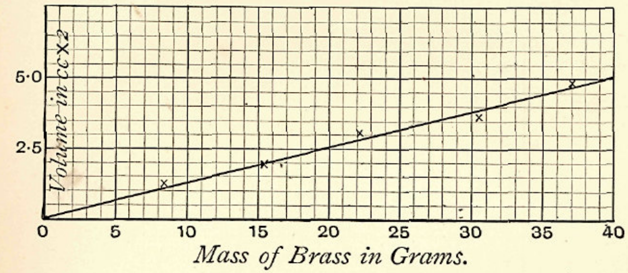


FIG. 98.—A diagram representing approximately the relation between the mass and the volume of brass, as obtained from a series of experiments. It will be seen that there are about 8 units of mass in each unit of volume. (Note each unit of volume takes *two* squares.)

the line joining these various points represents in a **continuous record the numerical values of both mass and volume.** That is, the records *of the two quantities* are made at once, and by the same mark.

It may be pointed out that we may shorten the phrase—the relation, or ratio, between the numerical value of mass and the numerical value of volume. For the sake of *rapid expression* we may speak of the relation between mass and volume, although we may be aware that, strictly speaking, there is **no relation between mass and volume possible. It is their numerical values which are related.**

153. Now it is not difficult to realize, and diagrams should be drawn to assist you to realize, that no change in the units by which you measure can prevent any one of these lines from showing what it is intended to show, namely,

First. **The lines exhibit the primary fact that the mass is proportional to the volume**; and

Secondly. They indicate **in what manner the mass and volume** (or rather, the numerical value of mass and numerical value of volume) **are related.** We may also point out that the lines must always be straight lines. *When one quantity is directly proportional to another a straight line alone can represent this fact.*

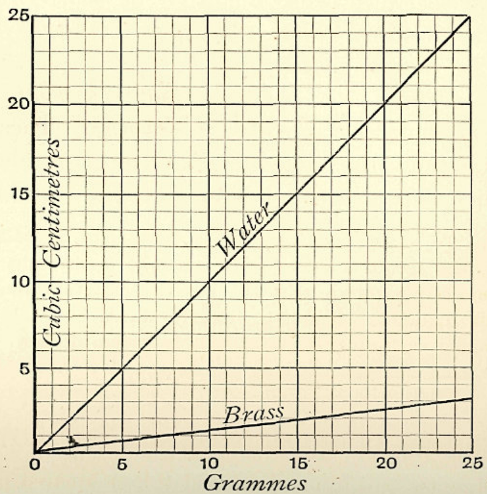


FIG. 99.—Diagram representing the relations between the masses and volumes of two kinds of matter, water and brass, and showing at the same time the difference in these relations. (Note that the scale for the volume of the brass is not the same as in the last figure.)

We may now proceed to take another step, by combining the two diagrams in one (Fig. 99). Here we

have represented in the same diagram the two items of information, which are given above, for two distinct kinds of matter. But we need not draw a limit at two lines. The same diagram may include any number of lines, each line representing these relations as they exist for each kind of matter.

154. In such a diagram we may give *any convenient value* of mass or volume to a unit of length, but **the relation existing between the lines themselves will always remain unchanged.** The same facts will always be represented. These facts are more easily expressed by using the word **Density,** which stands for the relation existing between the quantity of matter and the quantity of space which it occupies. We may say (1) *That the density of the same kind of matter under the same conditions does not vary*, and (2) *That different kinds of matter have different densities.* These statements express the same facts as are given above, and the lines shown in the diagrams are *lines of density*.

NOTE.—To avoid confusion, it is convenient to call the line forming the record a curve, even when it is straight. It is of the utmost importance that the meaning of every line and the scale of distances should be clearly stated in each diagram. Write under each diagram what it is intended to show, and write against each line the meaning you intend it to bear.

EXERCISES IN RECORDING CONNECTED QUANTITIES.

1. Draw two co-ordinates OL and OA on squared paper, and trace a curve which shall represent the relation between the magnitude of the side and that of the area of a square.

It is known that if the numerical value of the side of a square be 1, then its area will have a numerical value of 1; that if its side be denoted by 2, then its area has a magnitude of 4, and so on. To take examples, if the sides of a series of squares are represented by 1, 2, 3, 4, 5, etc. centimetres, then the corresponding areas will be 1, 4, 9, 16, 25, etc. square centimetres.

On your diagram make five crosses which shall mark distances in the direction of OL to correspond with the linear dimensions, and shall at the same time mark, in the direction of OA, distances to correspond with the square measures or areas. For instance, the centre of the cross a marks 1 unit of length in both directions; the centre of the cross b marks 2 units of length in the same direction as OL, and, at the same time, 4 units of length in the same direction as OA. Take further points c, d, and e, agreeing with linear dimensions of 3, 4, and 5, and also with the corresponding square measures.

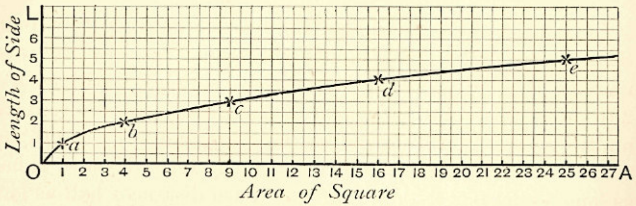


FIG. 100.—A curve showing the numerical values of the areas of various squares at the same time as it points out the lengths of the sides containing those areas. Hence it shows the relation between the linear dimension and the quantity of surface in a square.

It may now be found that these points a, b, c, d and e are too wide apart to trace a curve to include them with any certainty of accuracy. Points should now be interposed or *intercalated*, corresponding with linear dimensions of 1·5, 2·5, 3·5, and 4·5, and with areas of the squares of these numbers.

The curve connecting them may be traced by hand, but it may be drawn better by using a flexible lath, bending it to coincide with these points, and tracing a curve from its face.

We now possess a curve which exhibits the relation required. Test its accuracy by taking several intermediate points, measuring the distances in the direction representing linear dimensions, and also in the direction representing areas, and noting if they agree with your calculations.

2. Construct, in the same manner as in the last exercise, a diagram which shall exhibit the magnitude of the area of a rectangle, when one side remains unchanged, say at 2 units of

length, and the other side increases from 1, 2, 3, etc., up to 10 units. Remember that the area of a rectangle is calculated by multiplying together the two sides. Note that the curve expressing this relation is a straight line.

3. Construct a curve showing the relation between the area of a circle and its radius. Follow the instructions laid down in No. 1 as to the need of *intercalation*.

Remember that the area of a circle may be calculated from the square of the numerical value of the radius multiplied by a number 3·1416, or π; but since the square of the value of the radius is *always* multiplied by this number, the *relation between the areas* is not altered, and the shape of the curve is not affected, by omitting this number from the calculation.

In order to understand that the *relation* between two quantities is not affected by multiplying them both by the same number, it is only necessary to take two simple lengths, such as 2 ft. and 3 ft., repeat them two or three times, in other words, multiply them by 2 or 3, and we have 4 ft. and 6 ft., or 6 ft. and 9 ft. But each of these pairs is similarly related. 3 ft. is half as long again as 2 ft., and 6 and 9 ft. are distances half as long again as 4 and 6 ft. respectively.[1]

After drawing the required curve to express that the areas are always related in the same way to the squares of the radius, draw another curve on the same diagram showing the same relation, but in this case calculate the areas fully as $\pi \times r^2$. Note that the two curves are of the same kind (Fig. 101).

Note also that the curves are of the same shape as that expressing the relation between the numerical values of the side and area of a square (Fig. 100).

4. Make use of the squared paper to obtain a curve showing the relation between inches and centimetres. Take two lines, OA and OB at right angles (co-ordinates), and *calling* the sides of the larger squares (for we shall need subdivisions of these lengths) along OA, centimetres, and those along OB, inches, we can obtain a curve expressing the number of centimetres in any required number of inches. Note, *that lines may be made to act as signs of any quantities whatever.*

[1] Either an actual measurement or a pictorial representation will help you to realize this. A superficial knowledge is not enough.

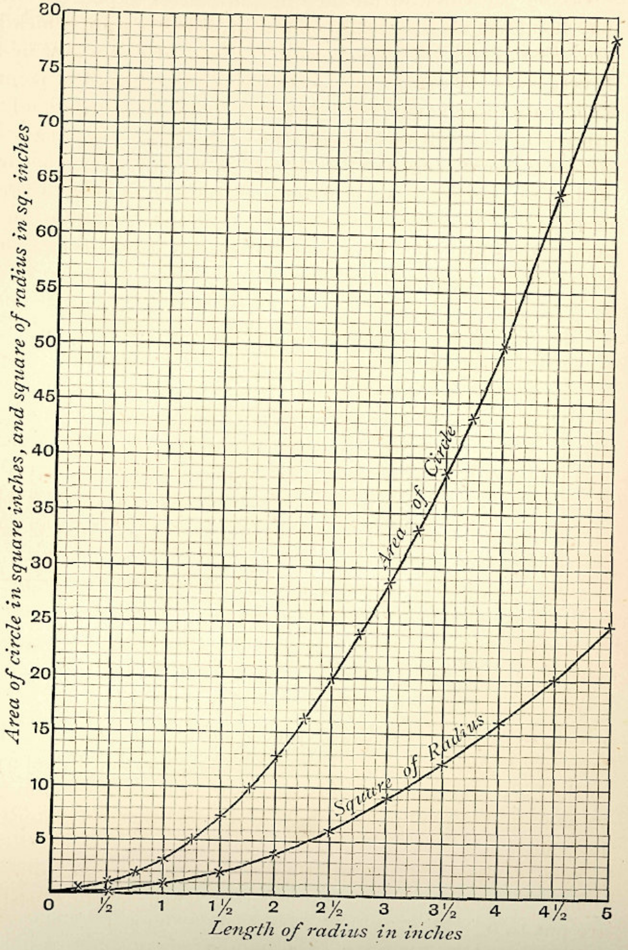


FIG. 101.—Two curves showing the relation between the radius and the area of a circle. One is obtained by calculating the area (by multiplying the square of the radius by 3·1416), and the other is obtained by considering the area as *proportional to* the square of the radius. The former may be the record of actual observations. It may be noted that the scale for areas is *one-tenth* that of the radii. In Fig. 100 the scales are alike.

Taking the value of an inch in centimetres to be 2·54, which may be measured practically on a scale, select a point which has 12·7 *divisions* in the direction of OA, and five *divisions* in the direction of OB (Fig. 102). We shall then have a point a which represents on our diagram both 5 inches and its equivalent in centimetres ($5 \times 2{\cdot}54$ or 12·7). We have *represented* equal lengths in each direction; but, since the units of length measured in different directions on our diagram *stand for different quantities* (in the one case standing for an inch, and in the other for a centimetre), the *actual length* along OB is not the same as that along OA.

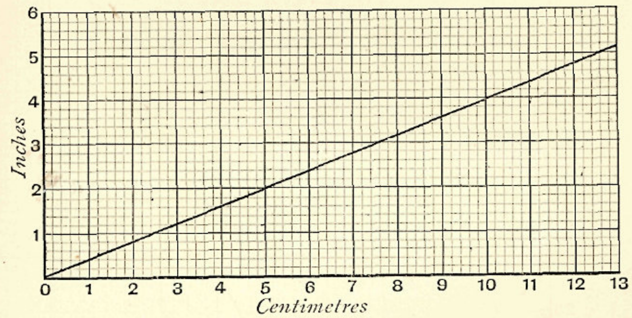


FIG. 102.—A diagram showing the value of lengths up to 13 centimetres, either in inches or centimetres.

Join the point so obtained with O, and we shall have a curve which represents all values from 1 up to 5 inches. We may take any length not greater than 5 inches, and find by referring to the curve the value of this length in centimetres. Similarly we may find the value in inches of any length not greater than 12·7 cm.

Such a curve does more than act as a record of observations. It affords valuable information when certain questions are put to it.

Note carefully that by producing the line we can compare the values of longer lengths in *the two systems of measurement.*

Find from your curve the value in centimetres of 3, 4, and 4·5 inches.

5. Make use of squared paper to obtain a curve showing the relation between pounds and kilograms.

Follow the same method again, and use the larger squares again. Suppose the units of length in one direction to represent pounds, and those in the other direction to indicate kilograms. Find a point which expresses the value of the kilogram in pounds. Join this point with the meeting-point of the two co-ordinates. (This point is called the *origin*.) Then this line may be continued to any distance, so as to represent any length within the dimensions of the paper.

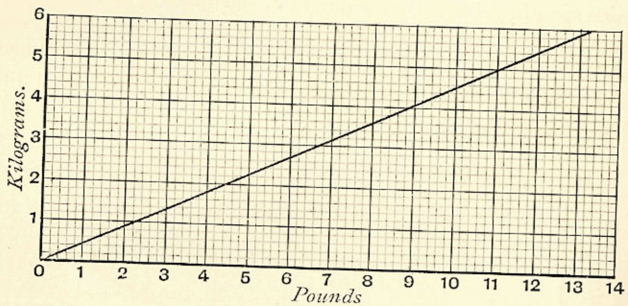


Fig. 103.—A diagram showing the value in both pounds and kilograms of masses up to 6 kilograms (or 13·2 pounds).

Find out from your curve the value in kilograms of 1·5, 2·5, 4 and 7 pounds, and also the value in pounds of 2, 4, 5 and 7 kilograms.

6. Find the mass of a flask which will hold 25 c.c. of liquid and is marked to show that volume. Fill it up to the mark with pure water and weigh again. Perform the same operation again in a 50 c.c. flask, and also in a 100 c.c. flask.

Make a diagram on squared paper which shall represent the results of your observations. Measure cubic centimetres along one co-ordinate and grams along the other. Connect the marks indicating the three observations by a line (usually called a *curve* even if straight).

Next make use of your curve to inform you of the mass of 10, 20, 60, etc., c.c.

The information given by the diagram should agree with

the results obtainable from observation. Test how far your curve correctly represents intermediate values by making observations. Weigh a beaker carefully, and then drop in from a burette 10, 20, 60, etc., cubic centimetres of water, weighing on the occasion of each addition.

Your observations should inform you that 1 c.c. of water contains 1 gram of matter.

Use a weighed beaker and a burette, and perform the same series of observations with methylated spirits, turpentine, ether, solution of common salt, and other liquids. Then draw the corresponding curves on a diagram, such as is shown for other substances in Fig. 104. The mass of 25 c.c. of mercury may also be found, and the masses of larger volumes be assumed. The beaker should be well cleansed and dried after each liquid has been used.

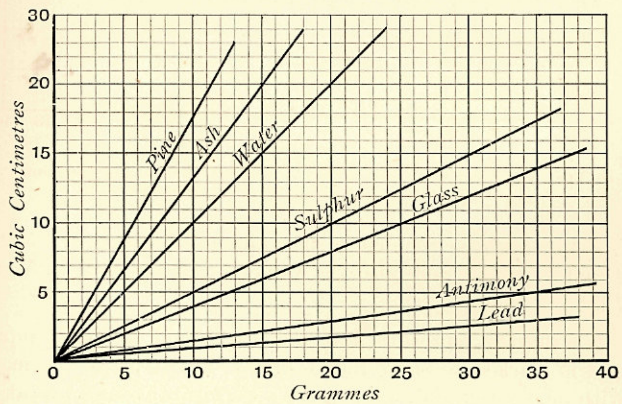


Fig. 104.—A diagram showing the relations between the volumes and masses for different substances, and also the relation between the masses of equal volumes of these substances, that is, their *relative densities*. For the latter purpose any horizontal line may be selected which cuts the various curves.

It may be admitted that *any point* on any of these so-called curves, provided the measurements expressed by *certain points* on them are correct, will express a certain fact, namely, the mass of a known volume of the substance. A given point may be selected, then its distance from the one co-ordinate will represent the numerical value of the volume, while its distance from the other will show the numerical value of the mass. *The points between those which record our observations should be of equal value as records of facts.*

The diagram will also reveal to us the masses of *equal volumes* of the various liquids. All that is needed is to draw a horizontal line from any convenient point on the line of volumes. Then the length, cut off between any one of the points of intersection with the curves and the horizontal co-ordinate, shows the quantity of matter contained in a given volume of the substance denoted by that curve. (One of the horizontal lines already existing in the diagram will serve the purpose equally well.)

In other words, the linear distances of the points of intersection, made by this line with the curves, from the line of volumes, stand for the relative quantity of matter in equal volumes of the various substances. These distances express the relative *densities* of the substances. And we may note that the density of water is 1.

7. Measure the mass of the given circular brass cylinder in grams, and then measure its volume by observing *how much water it displaces.*

Place water in a burette up to a certain level, say the 50 c.c. mark. Then slide the cylinder down the tilted burette. Note the level at which the water now stands. The difference of level enables you to measure the volume.

Now *calculate* the volume, by measuring the dimensions of the cylinder in cms., and multiplying the square of the radius by the height and by π ($\pi r^2 h$). The result should be the same as that obtained by observation of the displacement of water.

Knowing the numerical values of the mass and volume, ascertain the number of units of mass in one unit of volume. The number obtained is called the *density* of brass. This number may be regarded as an index of the compactness of the matter in that substance. There is more *matter* in 1 c.c. of brass than in 1 c.c. of water.

The great convenience arising from a knowledge of the number of grams of matter there are in 1 c.c. of a substance, lies in the opportunity of comparison which it affords. By

knowing the different quantities of matter, contained in the same volume of different substances, we can *compare their densities.* They are all brought to the same scale. In order to do this, it is always necessary *to divide the number of units of mass by the number of units of volume.* If we could always measure the masses of *equal volumes*, the numbers obtained would be *proportional to* the densities.

8. Measure the density of pieces of lead, iron, glass, and copper, by the method described in the last exercise, namely, by finding the mass and the volume in each case, and dividing the numerical values obtained for mass by those obtained for volume.

Use any graduated vessel, provided it admits of accuracy in reading volumes.

Prove by observation that the narrower the vessel the more accurate the results, for in a narrow vessel the *length* corresponding with a given volume is greater than in a wide one.

Make a list of the densities obtained.

9. Make calculations which will produce the numbers needed to indicate the densities of the above substances, if the unit of volume be changed to a cubic inch, and the unit of mass to an ounce. Notice that it is convenient to make two steps in the calculation :

(1) Find out how many grams there would be in a cubic inch, knowing that a cubic inch contains 16·38 c.c.

(2) Find out how many ounces there are in the number of grams found in the first part.

10. The density of mercury is 13·6. By measuring from two lines at right angles, on squared paper, construct a line which shall exhibit the mass of any volume between 10 and 20 c.c. of mercury. Then find out from measurement of lengths the masses of 12·5 and 16·5 c.c.

What would be the length of a column of water which would contain the same quantity of matter as a column of mercury of equal sectional area and 76 cm. in length ?

What would be the respective number of grams in 100 c.c. of water, brass, lead, and mercury ?

11. Ascertain the diameter of a circular tube (a burette, for

example), by allowing a column of water of measured length to be taken from it and weighed in a beaker. The beaker itself must be previously weighed, and the density of water is assumed to be 1.

(The *mass* of the water informs you as to the *volume* of the water. Then calculate the diameter of a cylinder of the measured height which is equal in volume to that of the water.)

12. You have given to you a piece of paraffin and a piece of brass for any investigations you may wish to make, and also a piece of paraffin which has embedded in it an unknown quantity of brass. The problem before you is to find out the mass of the imbedded brass without disengaging it.

THE MEASUREMENT AND RECORD OF DISPLACEMENT WITH REGARD TO TWO STRAIGHT LINES.

155. The simplest kind of change ever coming before us is change of position. It is of course necessary to be able to recognize or mark position, before we can perceive *change* of position; and when it happens that position is indefinite, a change in position may pass without notice. As a rule we mark in our minds the position of a body by observing those bodies which are near it. We refer to these bodies again if we wish to ascertain whether it moves. A rapid mental measurement takes place.

If, however, the position of a body at one moment may be denoted (in the manner already described) by a point in a certain position on a plane, and by another point to denote its position at some subsequent time, then we are on the way to learn something about the displacement of the body. If the positions of the body at small intervals of time could be measured, we should have them represented by points close together; and *in that case the line joining these points would serve as a record of the displacement of the body.*

Now we know that displacement may take place in a variety of directions, but we shall first deal with the case of a body being displaced in some direction in a given plane; for example, a stone moving on ice or a train moving through level country. It will be best to consider that we can measure the distance of the body from two lines at right angles. Let these two distances be marked, *on any convenient scale,* by corresponding distances from two lines drawn at right angles on squared paper. Let corresponding measurements be taken at subsequent periods of time, and marked on the paper. **We shall then obtain a series of points serving as a record of successive positions of the body.**

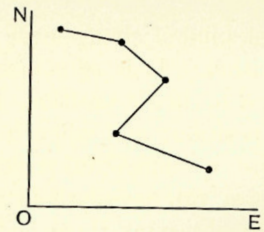


FIG. 105.—A curve showing the path along which a body has been displaced, or the successive positions of the body. The distances measured from the two lines *ON* and *OE* denote the position of the body at any moment.

If the displacement has been at all uniform, **the line joining these points will be a record of the path** along which the body has moved. It may happen that the displacement has been *very irregular,* and in that case the line is an incorrect record of the path. To make it correct, *the measurements must be made at intervals so small as to include or give an indication of all irregularities.*

156. The same result would be obtained if the body during its displacement showed its own path, or the line along which it had travelled. We should then *transfer this path to paper*, a line drawn to scale representing it. A body, for example, may be moving in a straight line to the N.E.; or in a circle of known radius from E. to W. by way of N. If the direction of the path be described in this way, measurements are not needed. They must have been made beforehand, for *they are implied in the description.*

It is an instructive exercise to classify the various paths in which a body may be displaced.

Divide them according to whether they form straight or curved lines; whether the bodies move in one plane; or whether they move in any direction in space. By means of a model representing three planes at right angles to one another an exact classification can be made and illustrated.

THE MEASUREMENT AND RECORD OF SPEED.

157. In addition to the record of the successive positions of a body, the **relation of these positions to time may be expressed.** We shall then obtain a value for the *magnitude of the displacement in a given time*, and this is what is meant by **the speed** or **rate of motion.**

A body moves slowly or quickly; and a comparison of *rates* can be obtained by measuring the change of position, and the time occupied in that change. It was found in the observation of the path of a moving body, that it was necessary to make a *number* of observations at small intervals, in order to be sure of the path of a moving body. So, too, it will be understood in the case of speed, that we must ascertain *the distance traversed in a large number of small intervals of time*, if we wish to gain an accurate knowledge of the rate of motion.

If it be known that the motion of the body is **uniform** and not irregular, then this course is *unnecessary*, **and any measurement of length traversed, together with that of time occupied, will give the value of speed.**

158. Two lines at right angles on squared paper may be used as a means of recording speed. The units of length, that is, of the distance traversed, are recorded along one line, and the time occupied in so doing is represented by a length measured along the other line. **An example of uniform motion will be found to be represented by a straight line.**

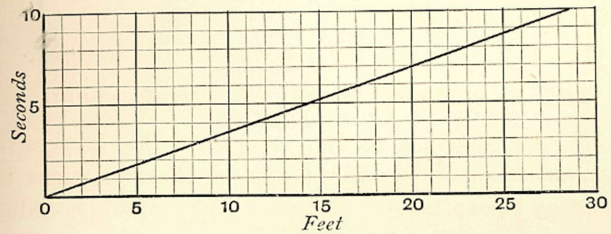


FIG. 106.—A diagram recording an example of uniform speed.

In order to compare speeds, it is necessary to determine the displacement of each body in the same interval of time.

When we say a speed is 50, two more things must be known before the statement can convey any true meaning to us. A speed of 50 certainly means *50 times the Standard length in the Standard interval of time*, but we must know what is the standard length and what is the standard time.

If, however, the measurement is understood to be made according to the Metric or Absolute System of Units, the statement would be complete.

159. There is another point to be noted. When the speed is irregular, its value can still be stated at any instant or at any point, by saying how many times the standard length would be traversed by the body in the standard interval of time, *if its speed at that instant or at that point were to remain constant throughout the standard interval of time.*

When we say that the speed of a train passing through a station is 50 miles an hour, we mean that, *if* the train continued to move for *a whole hour with the same speed as it had when passing the station,* then at the end of the hour it will have traversed 50 miles. We say this, moreover, when we may know as a matter of fact that the next stopping place is only 5 miles off.

We must try to look upon **speed as a property of a body which may last for an instant only,** and we can do so more easily by thinking of some cases of irregular speed, *e.g.*, the bob of a moving pendulum, or the piston-rod of a steam engine.

160. Now it will be remembered that the relative masses of equal volumes, or in other words, the *relative densities* of different substances, were represented by certain lines on a diagram (Fig. 104). In a precisely similar manner, we may represent on the same diagram **the relation between different speeds.**

Draw two lines at right angles on squared paper (Fig. 107). Along one measure distances to correspond with the lengths traversed in equal intervals of time, and measure along the other equal intervals

of time. By this means construct a curve to illustrate a uniform speed of 2 ft. a second. In the same way draw a curve to represent a uniform speed of 3 ft. a second. Then draw a horizontal line to cut both curves. The *relative speeds* will then be indicated by the *relative lengths* of the lines intercepted between the curves and the vertical co-ordinate.

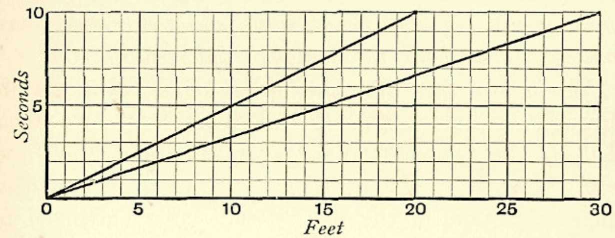


FIG. 107.—A diagram showing the relation between uniform speeds of 2 and 3 feet per second. All the horizontal lines exhibit this relation.

These lines serve the same purpose as those similarly described for densities. **In each case we obtain a relation between values which themselves express relations.**

EXERCISES IN THE RECORD OF VARIOUS CHANGES.

1. Plot a curve which shall describe the uniform growth of a person who increases 2 inches in height every year for six years. Assuming the person to be 4 ft. 6 in. at the beginning, find out from the curve his height after 2 years and 4 months.

2. Draw the curve illustrating the growth of a person who increases in 6 successive years 2, 3, 1, 3, 2, and 1 inches respectively. Assume the change in the rate of growth to be gradual.

3. Obtain the weather chart of an issue of the *Daily News* and the *Daily Telegraph*, and compare their modes of recording the height of the barometer. Observe that the curve of the

Daily Telegraph appears to be the more complete record. Is it really so?

4. Plot a curve to show the speed of a train running for ten minutes at the rate of 50 miles an hour; then becoming uniformly slower until, at the end of another ten minutes, it runs at the rate of 30 miles an hour; and continuing at this speed for twenty minutes, then pulls up to rest within half-a-mile.

5. Take from "Cassell's," or other Time-Table, the mileage and times of any train passing through a number of stations, and draw a curve expressing the distance and speed. Also draw the curve of the next train on that line. It is now required to run a special train between these two trains. Draw a curve midway between the two curves already drawn, and from that calculate the time at which it will pass through intermediate stations.

NOTE.—This method is used on railways in arranging traffic.

6. A train travels eastwards at the rate of 3 miles in 5 minutes, and northwards at the same time at the rate of 4 miles in 5 minutes. Draw a curve which will express its successive positions. In what direction must the line be laid to allow of this motion?

7. At what speed does the train in the last example really travel? Ascertain by measuring from the diagram.

8. A train is travelling in a direction N.N.E. at a speed of 50 miles an hour. Find out, by the use of squared paper, the distance it has travelled towards the east in 12 minutes.

9. Plot a curve which shall show a quantity a increasing at the same rate as a quantity b, that is, a quantity which varies directly as another. Also draw on the same diagram a quantity varying inversely as the other, that is, the quantity a growing smaller as the quantity b grows larger.

10. Plot a curve showing two quantities, a and b, varying inversely as one another, but varying in such a manner that $a \times b$ remains a constant quantity. This is a very important curve expressing certain important physical facts.[1] (Cf. Fig. 94.)

[1] For example, the relation between the pressure and the volume of a gas.

11. The curve exhibited below expresses the relation, which was found out by an actual experiment to exist, between the length and period of oscillation of a pendulum. Draw up from the curve a table which states the number of vibrations in a minute for pendulums of 10, 20, 30, to 100 cm. length.

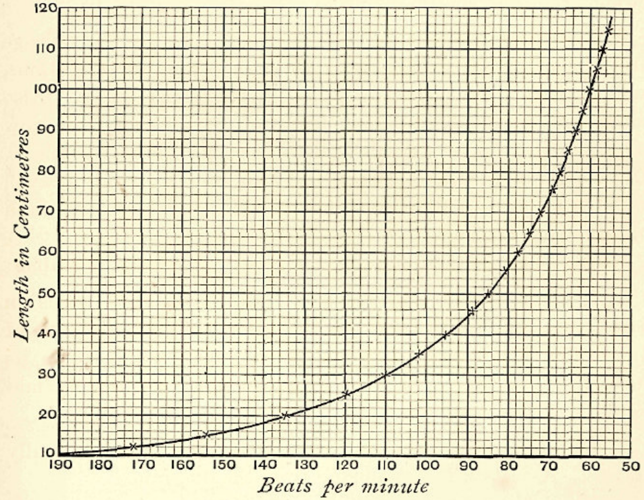


FIG. 108.—A curve showing the relation between the period of oscillation and the length of a pendulum. The pendulum is changed from 10 to 115 cm. in length.

SUMMARY.

161. We have learnt from the preceding exercises how to make a record of two quantities by the same sign. We can scarcely assert that the single record of two distinct quantities is made by a single operation; but the same sign or mark serves two purposes, and it does so by reason of its position on a plane.

We have already learnt that position on a known plane can only be described by means of two lengths[1] measured from two fixed points or lines. We have agreed that it is most convenient to describe the position of a point by means of its linear distance from two lines drawn at right angles. We have also seen that any kind of quantity may be expressed by means of lines, when they are drawn to a scale agreed upon beforehand.

Hence it is apparent that the position of a point may serve, by its distance from two lines at right angles, to indicate two quantities. And by a succession of points, or a line, we can exhibit the results of a number of observations of other quantities of the same kind. There is no object served in expressing in this way two quantities which have no connection. The chief service which these diagrams render is, that they form a record of two quantities which are presented by the same portion of matter, or by the same change.

We ought to be able to perceive that the idea of area is gained from the thought of measurement proceeding in two directions of space at the same time. The relation of areas to linear dimensions has been shown graphically. In a similar manner, the relation of the mass to the volume of a given body, and also the obvious fact that in bodies of uniform composition the mass varies directly with the volume, have been expressed by the same line.

A given point will express the magnitude of a given volume, at the same time as it records the mass

[1] It is true we may describe a position by saying that a point is a given distance along a line drawn at a certain angle with a fixed line, that is, by means of one length and an angle. But then the measurement of an angle requires *two lengths* to be measured.

of that volume; and a number of points can be placed in such positions as will show the masses of a variety of volumes of the same kind of matter. A line joining these points (assuming that a sufficient number of observations have been made, and that the matter is uniform or homogeneous) will form a continuous record of the constant relation which exists between these quantities in the kind of matter under observation.

162. But diagrams of this kind serve not only to collect the results of a set of observations, but also in many instances **to draw attention to errors of observation.** We can judge from the shape of the curve, whether it is a probable record of natural facts. We learn from the records of the most careful observers, and from innumerable repetitions of many measurements, that natural phenomena are not irregular and capricious; that connected quantities in nature are related in simple ways; and that there is a general tendency to uniformity. Hence the lines which represent these quantities and changes, must be **straight or evenly curved.** Irregular changes in direction of line would lead to inquiry, and the probable discovery of errors in observation.

But usually the points on the diagram merely admit of being **approximately connected** by a line. This must pass evenly between those points which cannot well be included in the line itself. That straight or uniformly curved line, which includes, or is nearest to, the largest number of points, is accepted as the most correct record. The deviations from that line may then be attributed to errors of observation which greater care may remove.

163. The diagrams, which have been drawn to represent relations of mass and volume, resemble those which may be employed to record the relations of length and time in the case of a moving body. The same remarks would hold true for each diagram, if we were to exchange the words *volume* and *mass* for *time* and *displacement*.

From each illustration of this method of record we learn how easy it is to perceive *a relation among relations;* or how easy it is to obtain a certain line, which enables us to compare different substances or different changes (Figs. 104 and 107) with one another, in regard to the relation which may be exhibited by the diagram.

From consideration of these facts we gather the necessity of less cumbrous phrases. We have found it convenient to speak of the relation of volume and mass, when we *mean* the relation of their numerical values; and now we need some shorter description than *the relation of mass and volume*, or *the relation of length and time*. The words *density* and *speed* serve this purpose, density denoting the relation of mass to volume, and speed the relation of length to time. Hence we speak of the **density** of a certain kind of matter when we wish to refer to **the number of times the unit of mass is contained** (or the fraction of that unit which is contained) **in the unit of volume.**

164. The density of a substance is denoted by a *number;* and the number tells us not only the number of grams in a cubic centimetre of a substance, but also the constant relation maintained between the mass and the volume of that substance. In the same way, the speed of a moving body is denoted by a number, which

tells us not only the number of times the unit length is traversed in the unit of time, but also the constant relation which is maintained between length and time at all points of the body's path, provided the motion be uniform.

The meaning of the term density (as derived from the results of two distinct measurements, and as expressing the ratio or relation between two numerical values) having become familiar, we can proceed to compare the density of one substance with that of another substance. We can also measure the change of density produced by change of physical conditions.

In the same manner the conception of speed being clearly grasped, the foundation is laid for the measurement of more complex quantities, such as rate of change of speed.

165. We may show by the following table the course which has been followed in the foregoing sections:

1. The relation between the numerical values of Length and Area.
2. The relation between the numerical values of Length and Mass, or Volume and Mass.
3. The relation between the numerical values of Length and Time.
   (1) Has increased our knowledge of certain figures.
   (2) Has introduced to us a new quantity, density, made up of two simpler quantities.
   (3) Has introduced to us a new quantity, speed, also made up of two simpler quantities.
4. The record, by means of diagrams exhibiting these relations, has shown that these relations may readily be expressed by lengths and, consequently, by numbers.

Finally we may add to this summary a warning, which was less necessary at the earlier stages, where observations and ideas have been of the simplest character. Now that we begin to touch on subjects comparatively complex, it becomes more and more urgent that care should be taken in our use of terms. It is a simple rule—**Never use a term or phrase unless you are quite certain of its meaning.** The word *density*, the word *speed*, for examples, must be signs of the same ideas whenever they are used. There should be no doubt nor confusion of thought over elementary conceptions; and a slower acquisition of knowledge at the beginning is better than an attainment which is too rapid to be enduring.

CHAPTER X.

MORE EXACT METHODS OF MEASURING LENGTH, MASS, AND TIME.

DISTINCTION BETWEEN CORRECT METHODS AND CORRECT RESULTS.

166. The examples of measurement which have been described in the earlier portions of this book have been selected for the *purpose of instruction rather than as illustrations of accuracy.* It is important that the **methods** of measurement, and the *reasoning* or *logic* underlying the methods, should be thoroughly understood before any attempt is made to enter into those *modifications* and *contrivances* which lead to **greater accuracy.**

It now remains for us to follow some of the plans which are frequently used in ascertaining the numerical values of quantities, of *length*, *mass*, and *time*, with more than the usual accuracy. These methods will not lead to *perfect accuracy*, but they approach it more closely than do the results with which we have been satisfied up to now.

The greatest accuracy is attained only after **many precautions** and **much care** have been taken. In addition to care and skill, elaborate instruments and contrivances are needed. Yet the results of measurements made under these conditions, however satisfactory and sufficient for the purpose in view, are merely *an approach to exactness.* **As more and more exact instruments are devised, inequalities which have been previously unobserved become more and more apparent.**

Frequent examples of this statement will present themselves in what follows, and it will be readily understood too, that inaccuracy is much more likely to be encountered in attempting to measure complex quantities, than it is in the case of the simple quantities, *length, mass,* and *time,* from which these quantities are derived.

EXERCISES IN MORE EXACT DETERMINATION OF LENGTHS.

1. Make two fine crosses on paper, and measure the distance between them. First apply a millimetre scale, holding it upright so that its divisions actually touch the marks. Make the exact centre of one mark coincide with a millimetre division, selecting one which is not at the end of the scale. Place the paper so that it presents a plane surface. Estimate by *eyesight* the fraction of a millimetre over, if the centre of the other

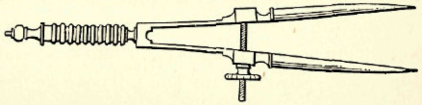
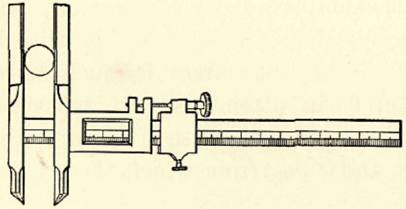


FIG. 109.—A form of spring-bows used in the comparison of lengths. The distance between the points of the spring-bows is adjusted until it is equal to the length to be measured. The spring-bows are then brought in contact with the scale or other length with which comparison is desired.

cross happens not to coincide with a division mark. Make several observations until you are able to estimate to eighths (·125) of a millimetre with accuracy. Next transfer the length, by means of spring-bows, or a good pair of compasses, to the scale, and read off the length on the scale, again estimating the value of any small length remaining, as the fraction of a millimetre.

2. Measure the length of a short piece of wire by applying it to a scale. Measure it again by using a screw-gauge, and also by means of callipers. Note that in each of these instruments the sense of touch tells us when the unknown length coincides with another length which may be read on a graduated scale. Take several readings with each instrument, and compare the average measurements.



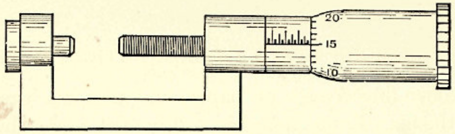


FIG. 110.—Callipers and screw-gauge. In the use of each instrument the sense of touch tells us when a certain length on the instrument corresponds with the length to be measured. The magnitude of the length on the instrument is then read off on the scale with which it is provided. The callipers may be used for *inside* or *outside* measurements. (Fig. is about ½ scale.)

3. Place the short piece of wire on a fine scale and support them under a microscope of low power, and look at the two ends of the wire, estimating their distance away from a division line. Or make one end coincide with a division mark, so that only one fraction need be read. Procure for the purpose *as fine a scale as possible;* but if you have not a scale with smaller divisions than millimetres, *estimate* as accurately as possible. If a microscope is not at hand, use a powerful lens. Compare the length thus obtained with your previous measurements.

It must be remembered in all measurements of lengths, areas, and volumes, that allowance ought to be made for alterations due to change of temperature. For this reason the standards (metre, yard, etc.) only exhibit their proper dimensions at a certain temperature, and comparisons must be made at this temperature. An investigation of these changes must come at a later period of our instruction, and for the present we must be satisfied with noting that change of temperature is a source of inaccuracy. At the same time it must be remembered, that any necessary corrections for change of temperature must come *after* we have made our measurements as accurate as possible.

4. Copy on a slip of glass a scale, 4 centimetres long, showing millimetre divisions. Clean the glass thoroughly with soda and hot water, and then coat it *thinly* with paraffin wax. Fix two thick needles firmly to a rigid rod. Fix the glass and an accurate scale in a line with one another on the bench-top by means of melted paraffin. Then placing one needle point in the centre of a millimetre mark on the scale, scratch the paraffin on the glass so as to lay the glass bare. Continue to mark successive millimetres in a straight line until you have 40 marks. Now rule a straight line through the wax, slightly above the marks, and extending from the first to the last mark. By means of a needle mark the centimetres by long lines and the millimetres by short ones of equal length, taking the marks already made as guides. Do this with the utmost neatness, taking care that the glass is exposed in such a way that a neat scale may appear when the glass is etched. *(This will be carried out by the master, with the aid of hydrofluoric acid.)*

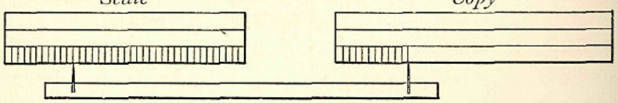


FIG. 111.—Diagram of the method of copying a scale by means of a rod carrying two fine points at its ends. As one point is moved through a given distance, as marked on the scale, the other point moves a corresponding distance on the object placed to receive the copy.

SUMMARY.

167. The exercises given above have been selected with a view to illustrate how far accuracy of linear measurement is possible, in cases where direct methods are alone available. These methods may be made to yield good results, provided certain requirements be kept constantly in view. The chief among these are the need of taking **repeated observations of the same measurement,** and the importance of exercising the **greatest care in each individual observation.** The first condition enables us to obtain an average or mean result, in which it may be supposed that, although some errors may have been repeated at each trial, yet some irregularities of reading have, at any rate, been balanced with one another; while, in order to obey the second condition, every precaution must be taken that common sense or ingenuity can suggest. We can now proceed to make use of those means of more accurate measurement, which are put into our hands in the shape of **delicate instruments** of skilful construction. We shall not avoid errors of measurement, even when using instruments of precision; but we can render them as slight as possible by making, with care, a large number of observations, and taking the mean of the results. For example, we may take seven readings of a length as follows:

3·25 cm. = 1st reading.		3·24 cm. = 5th reading.	
3·26 „ = 2nd „		3·26 „ = 6th „	
3·25 „ = 3rd „		3·25 „ = 7th „	
3·24 „ = 4th „			

Then 3·25 is the number showing the closest approach to accuracy attainable in centimetres.

THE VERNIER.

168. Divide a given line into *10 equal lengths.* Then take another line equal to *9 of these divisions,* that is, equal to $\frac{9}{10}$ of the first line. Divide this line into *10 equal parts.* We have now two lines, one of which is $\frac{9}{10}$ the other, and in addition each of its smaller divisions is $\frac{9}{10}$ of a division of the longer line.

If we take two strips of paper which have been divided in this fashion, and place them face to face, we can learn the principle of the vernier. It is best, however, to have a block of wood made to slide within guides, as shown in Fig. 112. The rectangular block should have, pasted on it, paper which has been divided into 10 equal parts. These should be together equal to 9 divisions of a scale which has been ruled on paper and pasted on one of the guides.

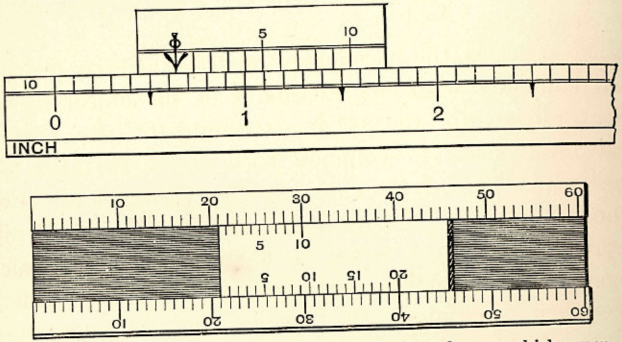


FIG. 112.—A form of model vernier, made of wood, upon which paper scales have been pasted, together with an illustration of a vernier reading to $\frac{1}{100}$ of an inch, and now showing a distance of ·64 inch.

With a board of this kind, about 18 inches or more in length, and with large divisions, the principle of the vernier is readily learnt. We may call the sliding block **the vernier.** Then each vernier division is $\frac{1}{10}$ smaller than a scale division. Therefore, if the *first mark of the vernier is made to coincide with the first mark of the scale*, there is left, between the first mark of the vernier and the first mark of the scale, **a small space which is $\frac{1}{10}$ of a scale division.** This should be illustrated on the model.

If the *second marks of each* are made to coincide, then a space of $\frac{2}{10}$, or *·2 of a scale division, is left* between the ends of the vernier and scale. If the *third marks coincide*, then *·3 is left*, and so on. When the *tenth marks coincide*, then a *whole division* is left, as might be expected from the fact that the whole of the vernier divisions are equal to 9 of the scale divisions.

169. The same mode of calculation may now be applied with *any division of the scale serving as a starting-point.*

In practice the vernier is moved until its proper end coincides with one extremity of the object to be measured, while the other extremity of the object has been made to coincide with the first mark of the scale. If, for example, this vernier-end now stands between the scale-marks of 6 and 7, it represents a length equal to **6 divisions of the scale, together with a fraction to be ascertained from the vernier.** We then look along the vernier, and fix upon *that mark which most nearly coincides with a division mark of the scale.*

It may happen that there is no *exact* coincidence. This will depend upon the size of the divisions. If mark 7 of the vernier *most nearly* coincides, we are then aware from our previous observations that the length over is ·7, and the whole length will be 6·7.

The advantage of the vernier lies in the fact, that it is far easier to observe the *coincidence of two marks* placed side by side, or to observe which marks out of a number *most nearly coincide*, than it is to *calculate* fractions of small lengths.

It will be perceived that *the accuracy of the reading* depends upon the *construction of the vernier and scale,* and in a rough model of this sort only approximate measurements are possible. But the principle is capable of being applied to smaller scales and of producing much more accurate results.

The vernier may have 20 divisions, together equal to 19 of the scale. This arrangement may easily be demonstrated by divisions on the other edges of the model, as shown. (Fig. 112.) In this case the vernier reads to $\frac{1}{20}$ of a scale division. Or there may be 30 coinciding with 29, in which case it reads to $\frac{1}{30}$ of the scale. **If the scale divisions are themselves very small, minute lengths can then be measured.**

The reading of the vernier on barometers, cathetometers, optical benches and spectroscopes should now be practised, together with the use of a lens for magnifying the scale.

EXERCISES ON THE VERNIER.

1. Construct a vernier and scale which will read to tenths of a centimetre by measuring off the necessary distances on two strips of paper. Test it by use.

2. Construct in the same way a vernier and scale which will read to $\frac{1}{32}$ of an inch. Test it by use.

It will not be necessary to have strips 32 inches long. Divide your scale into eighths of an inch. Make a vernier with four divisions equal to three of these eighths. This will enable you to read to $\frac{1}{4}$ of $\frac{1}{8}$, or $\frac{1}{32}$ of an inch.

3. Construct a vernier and scale reading to tenths of a millimetre. Make your division lines thin, and measure them accurately.

4. What kind of vernier would enable you to read to ·01 of a millimetre?

5. Where would coincidence be, with a vernier reading to tenths of a millimetre, when the length measured is 9·3 millimetres?

6. A circle is divided into 720 equal divisions. What divisions are needed on a vernier to enable you to read minutes?

Inspect the verniers on spectroscopes, theodolites or transit instruments afterwards.

MEASUREMENT OF SMALL DISTANCES BY MEANS OF A SCREW.

170. Small distances may be measured very accurately by means of *screws*. Everyone knows that a screw is so constructed that it advances or goes forward, when it is held firmly between the fingers, or by a piece of wood, and then turned round in one direction. That is, *the axis of the screw moves in a straight line when the head is turned round.* This follows from its construction. It has a spiral ridge cut upon it, called the *thread*, and if this be held tightly, the turning of the screw must bring about a displacement in a line corresponding with the central line or axis. The thread acts as a *constraint*, and **converts a circular motion into a rectilinear one.**

The thread of a well-constructed screw is cut so that the distance between any two consecutive turns does not vary. This distance may be made very small. In the case of such screws as are used for purposes of measurement, the turns of the thread are very close and appear to be straight across the screw. By tracing along several of its turns you will perceive that it is *continuous*, running on and on, forming a spiral, such as would be made by wrapping a thin wire tightly on a pencil. Such screws require great care in construction, and demand careful treatment.

For the purpose of measurement, a hollow spiral is cut out to correspond with and support the screw, and also to permit its movement to and fro without wear.

We then possess an instrument for measuring small lengths, called a micrometer screw.

171. The first measurements may be made by using a coarse screw supported on a stand, as shown in Fig. 113. The conversion of a *circular* into a *rectilinear motion* is readily seen, and we shall be able to understand how accuracy of measurement is obtained by the use of instruments of the same kind.

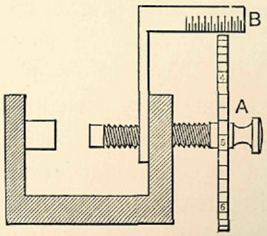


FIG. 113.—A model illustrating the use of the screw in measurement. *A*, the large circular head of the screw, is divided at its edge, so that a fraction of any turn may be measured. The number of turns is read from the scale *B*.

There will be a distinct gain in exactness, if we can *turn the screw through a small angle*, knowing at the same time the value of that angle. Notice that one complete turn of any screw may cause an advance,

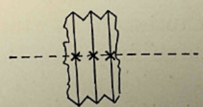


FIG. 114.—A diagram illustrating the change in the position of the same point on a screw as produced by one complete turn of the screw. The linear distance through which the point has moved is the same as the distance between two threads, provided it be measured in the direction of the length of the screw.

or linear displacement, equal to the *linear distance* between two threads. This arises from the form of a spiral. Make a mark in a screw fixed in a support,

and turn it once round. The mark then returns to the same horizontal level, *but it is one turn ahead or behind* as the case may be. Hence the whole screw has moved forwards, or backwards, through a certain distance, which is the same as the distance between two consecutive turns of the thread *measured in the direction of the length of the screw.* Now it is clear that a half-turn brings about one-half this change, a quarter-turn a quarter of the displacement, and so on. The diagram (Fig. 114) illustrates this movement.

172. All that is necessary then for great accuracy of measurement is *a fine screw* (or, as it is called, **one of small pitch**), and a large divided head attached to the screw. It is quite clear that the head should be made comparatively large, and that it should be divided, if we are to be able to move the screw through a small and known fraction of a turn. A large head, with 500 equal divisions marked in its rim, may be used; and the screw may have a pitch of half a millimetre. *Such an instrument would measure to the* $\frac{1}{1000}$ *of a millimetre.* It must not be forgotten, however, that we cannot avoid slight errors, especially in the case of very fine screws, caused by a certain "play" of the screw in the support, which is due to the need of freedom of movement.

The instruments which will illustrate the principle of measurement by means of screws should now be carefully examined. The ordinary wire-gauge may first be used. Then the spherometer, reading-microscope, and also micrometers from other instruments should be investigated, together with the means used for indicating the lateral distance travelled over by the screw. (These instruments are illustrated, and it is useful, even to an elementary class, to be allowed

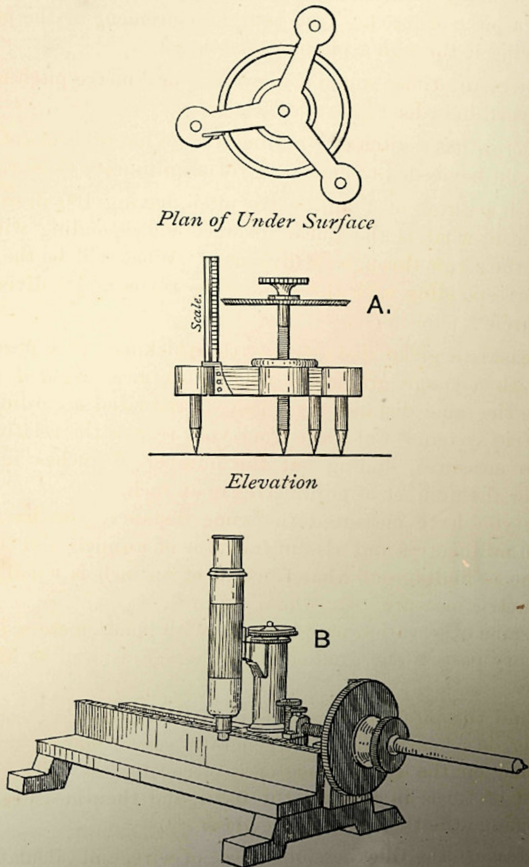


FIG. 115.—Illustration of spherometer, *A*, and reading-microscope, *B*.

to inspect such an instrument as a good reading-microscope.)

EXERCISES IN THE USE OF THE SCREW FOR MEASUREMENT.

1. Measure the pitch of an ordinary carpenter's screw, and state how many turns would be needed to drive it half-an-inch through a piece of wood. Test your measurement of the pitch by screwing it through a piece of card-board.

2. How many times would a screw of a millimetre pitch have to be turned in order to advance 1 inch?

3. A screw has a pitch of $\frac{1}{2}$ millimetre. What divisions of the head would be needed to enable $\frac{1}{100}$ of a millimetre to be read?

4. With a screw of 1 millimetre pitch, having 100 divisions on its head, what is the linear distance corresponding with a turn of the screw through 89 divisions? What will be the distance corresponding with three complete turns and 7 divisions of the circle?

5. Measure a given distance, say the thickness of a piece of wire, with a gauge graduated in English measure, and then measure the same distance by a gauge graduated according to the metric system. Calculate from your results the relation of the two measures, finding out the number of inches in the metre, or the number of millimetres in an inch.

Since you have measured the same distance, you have its value in millimetres and also in fractions of an inch, and this is the same as finding out what fraction of an inch is equal to a given metric measure. Or, a mm. $= b$ inch.

The same observation may be made with spherometers, if the laboratory possess one graduated in English as well as Metric measure.

6. Find the pitch of the adjusting screw of a microscope by focusing on a mark (dust will generally serve) on the bottom, and then on the top surface of a piece of glass. Count the number of turns in altering the focus, and then measure by a screw-gauge the thickness of the glass.

7. Measure the thickness of a piece of wire, and calculate the length of wire needed to be equal in volume to 10 c.c. Remember that vol. $= \pi r^2 l$, where $\pi = 3{\cdot}1416$, $r =$ radius (*i.e.* half the thickness), and $l =$ length.

8. Measure the thickness of a microscopic cover-glass, by

taking that of a number together by the gauge. Divide by the number taken, and compare the result with that obtained by using a spherometer, and measuring the thickness of a pile of them.

Notice that inaccuracies are probable in this method, through the layers of air and dust being measured together with the glasses; but that these are not so considerable as the probable inaccuracies due to the screw, when it is used in measuring one alone. The distance lost by looseness of fitting in the instrument may be a considerable fraction of the distance to be measured; while it may be inconsiderable when shared by 20 or 30, and consequently divided by these numbers. This loss of distance is diminished by turning the screw, in the *same direction as the measurement requires*, immediately before commencing the measurement.

9. Reverse the use made of the screw in previous measurements. Find out the length of a given straight line, by rolling the screw-head along it, and ascertaining the lateral distance by which the screw itself has passed through its support, while the latter has been prevented from turning. This gives the number of times the screw has been turned round; and all that is now needed, to know the length of the line, is the circumference of the screw-head. This multiplied by the turns, which are indicated by the lateral distance through which the screw has moved in its support, gives the length. Gauges, spherometers, and the instrument described may be used in this way.

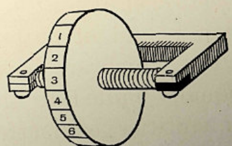


FIG. 116.—A form of instrument by which a given length may be determined from the number of turns taken by a wheel in travelling along the length. Since the wheel moves along a screw at the same time, the number of turns is indicated by the change of position of the wheel on the screw.

A rough *opisometer* (or wheel-measurer) may be made to give good results by fixing a wheel with a graduated rim on a screwed axle, so that as it rotates it goes forward along the screw.

Notice that in such an instrument, as in the reversed use of the instruments described above, it is the movement of the support, or the hollow screw, over the surface of the screw, that is marked, not the progress of the screw through the support. An instrument for this purpose, which may be easily constructed in the workshop, is shown in Fig. 116.

MORE EXACT MEASUREMENT OF MASS.

173. We have already learnt how to use the balance, an instrument which enables us to judge, very readily and very certainly, when *two quantities of matter are equal*, that is, to perceive when one quantity of matter produces the same effects under certain conditions as another quantity. It must always be remembered that this is the aim in the operation of weighing—**to find out when two quantities of matter are equal in a certain respect.** They may be very unequal in other respects. We put aside from our thoughts all difference of appearance and other properties, **in order to investigate one special kind of equality.**

At present, our knowledge of the subject, and our capacity for accurate thinking, only admits of that equality being understood as *an equality in the extent to which they are pulled or attracted to the earth.* A pound of lead and a pound of butter are two bodies of *very different nature*, which are, however, *equal in one way.* They are equally pulled to the earth, and we call them equal quantities of matter. Until more progress is made in our observations of nature, we must be content with this description.[1]

[1] When that progress has been made, we shall understand that they are also equal with regard to the transfer of something called *Energy*, or, in other words, that the same *effort* produces the same result on them, and this holds true whether they are on the surface of the earth or elsewhere.

174. There is more, then, in the act of weighing than merely adjusting weights on one pan until they *balance* a body placed on the other pan. **We are finding out how much matter, measured in terms of a known standard, is required to produce the same effect as one unknown quantity of matter produces.** The balance is the instrument which enables us to perceive when these effects are equal, by allowing them "to balance" one another.

To take a simple illustration, a body is weighed. It is said *to be* 4 grams. We mean that it contains 4 grams of matter, or 4 times a standard quantity which we call a gram. We base our statement on *the position or motion of a pointer, which indicates* a certain evenness in the beam and pans of the balance. This evenness is produced by the matter in the two pans being equal in quantity. Each pan with its contents is equally pulled to the earth. Hence arises the equilibrium or balance.

We may express the fact differently by saying, that **the two quantities of matter behave alike under the same conditions, and hence we call them equal.** One of the quantities is made up of multiples and fractions of the standard mass previously constructed. But it is clear that we *depend upon the balance* to tell us *when* the conditions are equal. If it has been correctly made and tested, we may assume that the two pans do provide "the same conditions," and that level beam or even swing of pointer do show *equal masses producing equal results.*

175. But it is very probable that the instrument which you are using, although it may be accurate when compared with the ordinary balance used in

shops, is yet only capable of denoting *approximate equality* even when you adopt the method of substitution which was described in section 31. **A more sensitive balance may show that they are not quite equal.** A still more sensitive balance may reveal even further inequalities; and we can easily understand that the search for equality in this, *as in all measurement of quantity,* may be indefinitely defeated.

But we can assert that *we are within a certain distance or within a certain limit of equality.* With a good balance, such as is too delicate for ordinary use, we can be certain that two quantities of matter are equal within $\frac{1}{10}$ milligram. With a balance, such as a student may use, you can "weigh," or find equality with the certainty of not having a greater difference between the bodies than 1 milligram of matter. There are not many investigations in which you will need a nearer approach to exact equality than the balance can show.

After a time, we shall find the need of some means of *marking out* and *separating* even smaller quantities than $\frac{1}{10}$ milligram. Yet, there is nothing which dispenses with the use of the balance. That is still an essential instrument, but we can *take the results of its use as the basis of further measurement and division of matter.* The following exercises illustrate this method.

METHOD OF MEASURING MASS BY SOLUTION.

176. The balance can inform us when two quantities of matter are equal within a very small quantity indeed. We are able in consequence to state how many times a given body contains the standard quantity of matter. This is the object of weighing. The balance may, however, be adapted to further purposes. We may procure a long length of wire, of uniform section (*i.e.*, any section or cutting of which would present the same area), and also of uniform composition. In such a case we might find out the quantity of matter in the whole wire, measure its total length, and then divide the wire into *known fractions* of that length. **These fractions of the whole length would contain corresponding fractions of the whole mass.**

For example, if 2000 cm. weighed 1 gram, then 1 cm. of the wire would weigh $\frac{1}{2000}$, or ·0005 gram. We could cut off *any required mass by measuring off the length corresponding with that mass.* It would be easy to subdivide the wire into desired fractions, assuming that its uniformity be known; and it would also be easy to obtain by this means a very small quantity of matter, one smaller indeed than the balance could weigh.

But both these results, namely the *minute subdivision of a given quantity of matter and the value of those subdivisions in terms of the standard,* can be obtained in a very convenient manner if the substance be soluble in a liquid.

177. In the process of solution a substance enters into and combines with[1] a liquid **so completely that we can no longer distinguish between them.** One substance is lost, as it were, in the other. They cannot be separated by hand, and they cannot be distinguished by the eye. *In fact they cease to be separate bodies.* The change is a gradual one, and may take in some

[1] You will probably have to use the word "combine" in a special sense, as referring to chemical combination, when you begin to learn chemistry.

cases a considerable time. In other cases it may not be *complete, a part only entering into solution.*

The liquid which acts as a **solvent** towards the largest number of substances is water, and water is the liquid with which we shall make experiments. A certain quantity of common salt is added to water. In a short time it disappears. We obtain a solution of common salt in water. It may be proved that this solution, after the lapse of some time, or after being well stirred, contains the salt **evenly distributed throughout the water.** Upon this fact hangs the success of the method, and it must be clearly understood **that the same volume contains the same quantity of matter.** This can be proved with the utmost certainty by practical means (which are not suitable, however, for repetition by beginners), and it has been so proved by innumerable tests in the past.

178. Putting confidence in these proofs we can assert, *that a given fraction of the total volume of the liquid contains a corresponding fraction of the total mass of the substance,* just as we were able to assert above, that a fraction of the total length of the given wire contained the same fraction of its total mass. If 1 gram of common salt be dissolved in 1000 c.c. of water (*or more accurately,* if we dissolve 1 gram of the salt in water, then make up the volume to 1000 c.c. and thoroughly mix the liquid), we have distributed the salt *evenly* through a volume of 1000 c.c.

Now by placing some of the solution in a burette, which is graduated in fifths of a cubic centimetre, it is easy to deliver $\frac{1}{5}$, or ·2, of a cubic centimetre of the solution, and therefore to obtain a quantity of liquid containing $\frac{1}{5}$ of $\frac{1}{1000}$ of 1 gram of the salt. **That is, we can make sure of obtaining as small a quantity as $\frac{1}{5}$ of a milligram, even when as large a quantity as one gram is made the starting point.**

But a balance gives correct information about much smaller quantities than a gram. A quantity no larger than a milligram could be made the subject of subdivision; and in this case we can obtain and deal with a quantity of salt no larger than $\frac{1}{5}$ **of** $\frac{1}{1000}$ **of** $\frac{1}{1000}$ **of a gram, or ·0000002.** It must not be forgotten that delicate manipulation is needed in order to gain such results. The illustration (Fig. 117) indicates the method in a manner.

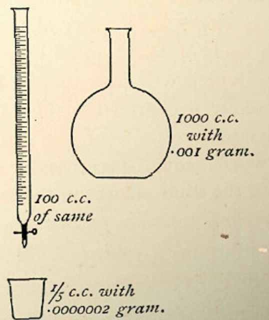


FIG. 117.—Illustration of the method of measuring mass by measuring the volume of a solution containing it. The given mass is held in solution in the flask, and by means of the burette $\frac{1}{5}$ of 1 c.c. of solution can be delivered into the beaker.

EXERCISES IN EXACT MEASUREMENT OF MASS.

1. Five grams of a substance are dissolved in 1000 c.c. of water, how much of the solution will contain ·5 gram?

2. Two cubic centimetres of solution contain ·05 of a substance, how much will there be in 1000 c.c.?

3. If 50 c.c. of solution contain ·1 gram, how much will there be in 105 c.c.?

4. How much salt will there be in 125 c.c. of a solution containing ·2 gram in 800 c.c.?

5. 1000 c.c. contain ·2 gram, how much solution will contain ·005?

6. What mode of solution would enable you to obtain ·00001 of a solid?

7. How would you dilute a solution containing ·5 gram in 1000 c.c., so that it would contain ·4 gram in 1000 c.c.?

8. How would you dilute a solution containing ·12 gram in 1000 c.c., so that 1 c.c. might contain ·00004 gram?

9. What length of wire would contain ·0005 gram of matter if 1 yard weighed 1 gram?

10. Weigh some dry sodium carbonate, dissolve it in a known volume of water. Place some of the solution in a burette. Next take a weak solution of sulphuric acid and place three volumes of 25 c.c. each, by means of a pipette, in separate beakers. Add to each of these equal quantities of acid a small quantity of *litmus*, which will turn them a bright red.

Note that the colour is changed to blue (showing the acid to be *neutralized*) by equal volumes of the sodium carbonate solution, or in other words, **equal quantities of sodium carbonate are required to produce the same change in the same substance.**

NOTE.—*A variety of exercises of this kind may be carried out with a view to practice in this method of calculation of quantities. It is known as* **volumetric analysis.**

GENERAL METHOD OF DETERMINING DENSITY.

179. In order to determine the density of a body, that is, the relation between its mass and volume, it is necessary to find out the numerical value of its mass and that of its volume. These are the two requisite measurements which have been already described. When they have been made, we can proceed to find *the number which expresses density.* For example, if the mass of a body be 12 grams, and its volume be 3 cubic centimetres, then $\frac{12}{3}$ or 4, expresses its density. **The**

number 4 tells us that each cubic centimetre of that kind of matter contains 4 grams of matter.

If we make the same measurements of *another portion of the same kind of matter*, the same result will be obtained. If, for example, we weigh a portion of that matter which has been found to measure 7 c.c. in volume, it will be discovered that it contains 28 grams of matter. This must be so if we are measuring the same kind of matter, for $\frac{28}{7}=4$, and there must be 4 grams in each cubic centimetre.

180. The number 4 is here intended to convey to us the information, that the relation existing between the mass and volume of this kind of matter is such that 4 grams of matter are contained in 1 c.c. of volume. The advantage of dividing the number of units of matter by the number of units of volume, and so obtaining the quantity of matter in the unit quantity of volume, lies in **the opportunity it affords for comparison.** If we know the quantities of matter in 1 c.c. of several bodies, we can compare these bodies with one another on an equality. *The numbers show at a glance whether there is much or little matter in a known volume.*

It is clear that *if we changed our standards* we should obtain *other numbers for densities*, but the comparison of these numbers with one another would show that they express the same facts of nature. If we were to use *pounds* and *cubic inches* as standards to measure all bodies, we should still learn from the numbers expressing densities that in one substance matter is more tightly packed, so to speak, than it is in another. We should also learn *how much more* tightly it is packed.

181. The value known as density being derived from **two measurements**, namely, those of **mass** and **volume**, it is clear that accuracy in making these measurements is needed, before we can obtain correct results for density. There is no difficulty in obtaining, as accurately as we need, the numerical value of mass; the balance can inform us as to the mass of a body with very considerable accuracy. **The measurement of volume is not so easy.** If a body has *a regular shape* such as *a sphere, cube, cylinder*, etc., we can calculate its volume from the linear measurements which may be made upon it. Provided the body is large enough to permit these measurements to be made correctly, the volume calculated from them will be correct.

But these conditions can happen only occasionally. **Volumes have to be measured, as a rule, indirectly,** and the chief method employed is to allow the body, of which the volume is to be measured, to be immersed in a liquid contained in a graduated vessel, water as a rule being used. **We ascertain the volume of the body by the alteration of the level of the liquid, due to the body occupying some of the space which was previously occupied by the liquid.**[1]

But this volume, after all, is not a quantity which can always be ascertained with certainty. Experience has shown that a slight error in reading a *level* may mean a considerable error in the *volume* corresponding with it. A narrow burette carefully graduated and read with care, may give for small bodies fair results. But greater accuracy is gained by an entirely

[1] In other words, we make use of a property of a liquid which enables it to take any shape which may be forced upon it. An irregular shape (that of the body) has been *translated* into a regular shape, and the volume of the body is measured by the volume of a regular-shaped body of water, that between the old and the new level.

different plan, that of **weighing the body while it is immersed in water.**

182. In order to understand this method, several important observations will have to be made; and they are best taken in the order here given.

1. Weigh separately several bodies of known or measurable volume, with fine fibres of silk or fine wire attached. For example, select several pieces of brass or iron of different volume. If they are not of known volume or regular shape, allowing the volume to be calculated, proceed to measure their volume by immersing them in water in a burette or narrow graduated cylinder. *Bodies which differ considerably in volume should be selected.*

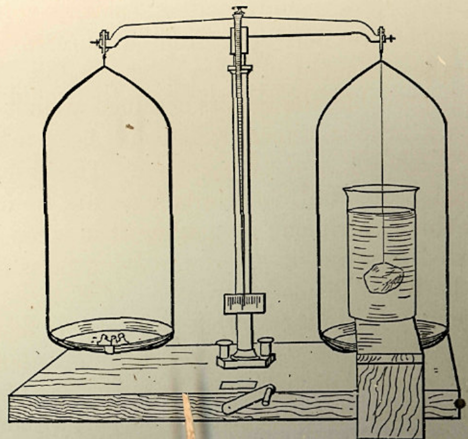


FIG. 118.—A body immersed in water in the manner shown is counterpoised by a smaller mass than when not so immersed.

Now, suspend these bodies in turn to the hook of the balance, as shown (Fig. 118), so that they may be

immersed in a beaker of water, placed underneath in such a way that the operation of weighing is not interrupted. Notice that these bodies are now counterpoised by masses in the other pan, which are smaller than were previously required. In fact, the water *supports* the bodies.

Take down the *difference* in the weights required in each case for counterpoise, and note **that they are proportional to the volumes of the bodies.** *The support rendered by the water is proportional to the volume immersed.*

On further investigation of *the difference observed*, you will find that it represents in each case **the mass of the water displaced by the body.**

Make certain of this statement by weighing successively, in a counterpoised beaker, volumes of water which are delivered from a burette and are equal to the measured volumes of the bodies.

If, as is probable, the results obtained are not quite in keeping with the statement which has been made, it is through *inaccurate reading of volume.* The observations should be repeated until concordant results are obtained.

From this experiment we gather that when a body is immersed in water, it *appears to lose* a mass equal to that of the water displaced by it.

2. We may next observe that the support or upward pressure given *to the body by the water* is balanced by an equal downward pressure *received by the water itself.* **The water does not give any support without being itself changed.**

A quantity of water in a beaker, not too large, is counterpoised on the balance, and then each of the bodies used in the first experiment is hung by a support independent of the balance, as shown (Fig. 119).

It will be found that in each case an additional counterpoise is needed, and that the water "weighs" *as much more*, in each case, *as the object appeared to lose* when it was weighed in the manner described in the first experiment.

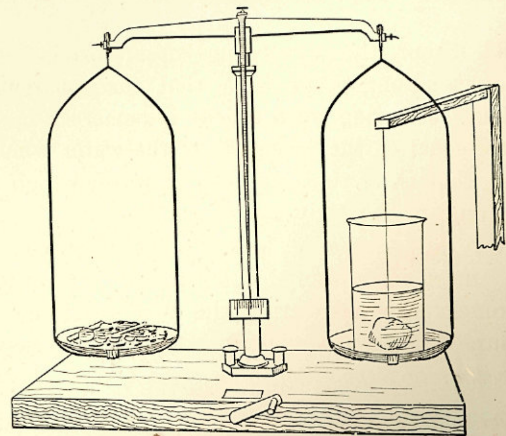


FIG. 119.—The counterpoise needed for a given quantity of water in a vessel is increased by immersing a body in the water in the manner shown, although the body does not hang from the balance.

3. Lastly, we may demonstrate the same facts by counterpoising a beaker of water and an object, *first*, when placed side by side in the pan of a balance, and *secondly*, when the object is suspended from the end of the beam, and at the same time immersed in the water. **The counterpoise is maintained.** That is, when both the water and the object are weighed together there is no difference in the total weight made by suspending the body in water. *The downward "pull" or "weight" is the same, however we may alter the arrangement on the pan itself.*

This statement may be presented as a general truth,

although this observation alone ought not to be accepted as a sufficient foundation for it. **The total mass of a system of bodies[1] cannot be changed by any alteration in the relative position of the bodies, nor can the effect of that mass on bodies outside the system be altered by any changes within the system itself.**

183. From the preceding observations we have gathered, among other facts, that **the apparent loss of "weight," occurring when a body is immersed in water, is exactly equal to the "weight" of the water which it displaces**, that is, of that quantity of water which is equal in volume to the body itself.

Now the *density of water* is well known. It has been most accurately determined. Therefore the difference observed during the two weighings gives a measure of the *volume* of the body, for it is easy *to find the volume corresponding with a given mass of water*.

Several observations should now be made, and the results entered as follows:

Determination of the density of a body.

Mass of body	=22·753	grams.
Apparent mass when immersed in water	=19·798	,,
Mass of water equal in volume to the body	= 2·955	,,

Now, the gram was made as exactly as possible equal to the quantity of matter contained in 1 c.c. of pure water at 4° C., and hence 2·955 grams of water occupy 2·955 c.c. of volume[2] approximately.

Hence the mass of the body is 22·753 grams, and its volume

[1] By the word "system" we describe any collection of bodies which are being jointly investigated.

[2] This statement is not strictly true, but it may be accepted for the present. The balances you use will probably not be sensitive enough to detect that it is not true, if you proceed to weigh the mass of various volumes of *pure* water.

is 2·955, and its density is obtained by dividing the numerical value of mass by that of volume, or

$$\frac{22·753}{2·955} = 7·7 \text{ approximate density.}$$

We may next point out that the temperature of a body affects its density. As a body becomes warmer it expands, *and its density diminishes.* Corrections for temperature should always be made, or the results of our experiments will not be trustworthy. But these details must be left to a later period.

ADDITIONAL EXERCISES IN DETERMINATION OF DENSITY.

1. Weigh a flask, such as that shown in Fig. 120 (Exercise 2), fitted with a ground stopper, through which a small hole passes vertically.[1] Fill the flask with a liquid, and fit in the stopper.

The small opening allows the excess of the liquid to escape. Wipe away any liquid outside the flask, and proceed to weigh.

Use in succession the following liquids, carefully cleaning the flask after each operation:—pure (*i.e.*, distilled and boiled) water; a solution of common salt, as strong as can be obtained (a *saturated* solution); ether; methylated spirits; benzene, and turpentine.

The numerical values obtained for the masses will be in the same ratio as the densities of these substances, for we have measured the mass of the same volume of each, and we have measured under the same conditions.

If the density of pure water at the temperature of the room be known, it will be easy, first, to calculate the volume of the flask, and, second, to find the density of the other substances.

Write out your results as follows:

Mass of flask when empty	= 5·953 grams.
Mass of flask filled with pure water	= 55·903 „
Mass of water	= 49·950 „

Assuming density of water at temperature of room to be ·999, or that 1 *c.c. of water contains* ·999 *gram,*

$$\text{since } \frac{49·950}{·999} = 50,$$

then the flask contains 50 c.c. of water, and its capacity is 50 c.c.

[1] This may be replaced by a well-fitting india-rubber cork, through which a small channel in the side has been cut.

Mass of 50 c.c. of salt solution	$= a$,	Density	$= \frac{a}{50} =$
„ „ ether	$= b$,	„	$= \frac{b}{50} =$
„ „ methylated spirit	$= c$,	„	$= \frac{c}{50} =$
„ „ benzene	$= d$,	„	$= \frac{d}{50} =$
„ „ turpentine	$= e$,	„	$= \frac{e}{50} =$

2. Making use of a flask of the same kind as that described in Exercise 1, find out the density of a powder.

The volume of the powder is ascertained by finding *the mass* of the water, which is needed to fill the flask when it contains the powder.

From *the mass* of the water we can *calculate its volume*, and subtracting this volume from the total capacity of the flask, we obtain the volume of the powder. The total capacity of the flask, if not known, can be found by weighing pure water in it, as described in Ex. 1.

In other words, the powder and water together fill the flask; and we can calculate the volume of the water from its mass, which is easily measured by weighing the flask with the powder alone in it, and then filling up with water and weighing again.

Enter as follows:

PART A.

(1) Mass of flask	= 5·953 grams.
(2) Mass of powder and flask	= 11·968 „
(3) Mass of powder, flask, and water	= 58·924 „

From (1) and (2)

Grams.	
11·968	
5·953	
6·015	= mass of powder.

From (2) and (3)

Grams.	
58·924	
11·968	
46·956	= mass of water required to fill flask with powder in it.

PART B.

(4) Mass of flask, when cleaned and completely filled with pure water } = 55·903 grams.

From this we can now obtain the volume of the powder. For knowledge of *mass* enables us to calculate *volume*, provided *density* be known.

From (4) and (1) we obtain total capacity,

Mass of flask full of water	= 55·903 grams.
Mass of flask alone	= 5·953 "
Mass of water	= 49·950 "

The density of water at the temperature of the room being assumed to be ·999, *i.e.*, 1 c.c. of water contains ·999 gram, then

$$\frac{49{\cdot}950}{{\cdot}999}=50,$$

in other words, there are 50 times ·999 gram *in the total mass* of water, and therefore there must be 50 c.c. of volume.

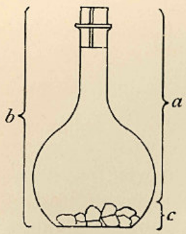


FIG. 120.

By means of the same reasoning we can find the volume of the water obtained in Part A. This is the water required to fill the flask when the powder is already in it. It was found to be 46·956 grams. Therefore

$$\frac{46{\cdot}956}{{\cdot}999}=47{\cdot}003$$

is the volume of this water.

We now proceed to find the volume of the powder:

Capacity of flask	= 50·000 c.c.
Volume of water in flask together with powder	= 47·003 c.c.
Volume of powder	= 2·997 c.c.

Therefore the mass and the volume of the powder being known, we obtain its density,

$$\frac{6{\cdot}015}{2{\cdot}997}=2{\cdot}007, \text{ density of powder.}$$

A diagram may render clearer the operations which have just been described (Fig. 120).

(1) The mass of *b* taken from the mass of *b* and *c* yields the mass of *c*.

(2) The volume of *a* taken from the volume of *b* yields the volume of *c*.

From these the density is obtained.

3. It may sometimes happen that the body, of which the density has to be measured, is not *heavy enough* to sink in water. In this case we must modify the process. The body, such as wood or wax, is made to sink by fastening to it a sufficiently heavy body, such as a piece of lead, of which the mass and volume have been previously determined. An example will illustrate how to proceed.

Mass of light body alone	= 4·5 grams.
Mass of lead alone	= 34·3 grams.
Apparent mass of both together when immersed in water	= 29·95 grams.
Difference between their joint mass in air and in water, which represents the mass of the water displaced	= 8·85 grams.
Consequent volume of water displaced by both together	$=\frac{8{\cdot}85}{{\cdot}999}=8{\cdot}859$ c.c., about.
Volume of water displaced by lead alone	$=\frac{34{\cdot}3}{11{\cdot}2}$ (density of lead) $=3{\cdot}062$ c.c., about.
Volume of light body	= 8·859 − 3·062 = 5·797 c.c.
Density	$=\frac{4{\cdot}5}{5{\cdot}797}={\cdot}77$, about.

4. Compare the densities of any of the liquids used in Exercise I., by means of weighing the same body when immersed in them.

We have learnt that when a body is suspended in water it is supported by the water. It *appears* to lose a mass equal to that of the water which it displaces. If we substitute another liquid for water, the body is supported, *but to a different extent*, and the extent varies with the density of the liquid.

As might be expected, the support experienced by the body is that which previously sufficed to support that portion of the liquid which is now displaced by the body. The following diagram illustrates this statement (Fig. 121).

In making our observations we shall displace *equal volumes of different liquids;* and the difference between the counterpoise required when the body is weighed in air, and that required when the body is weighed in a liquid, gives the mass of that volume of liquid. (Refer to previous exercises.)

It might be surmised that the air itself supports bodies. This it does to some extent; but we cannot yet allow this to be taken into consideration, although it must be done if we are to measure mass with *the greatest accuracy.*

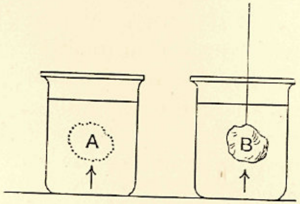


FIG. 121.—*A* is a portion (equal in volume to the body *B*) of the liquid in a vessel maintained in its position by the rest of the liquid. *B* is a body receiving the same support as the liquid *A* which it displaces.

Use a large glass stopper, weigh in the air, and then in each of the liquids in succession. Enter your results as follows:

Mass of body in air	= ..			
Mass of body in water	= ...	Difference	=	...
,, ,, salt solution	= ...	,,	=	...
,, ,, ether	= ...	,,	=	...
,, ,, spirit	= ...	,,	=	...
,, ,, benzene	= ...	,,	=	...
,, ,, turpentine	= ...	,,	=	...

Now if the difference obtained in the case of water be divided by ·999, we obtain the volume of the water in cubic centimetres. The masses of the same volume of the other liquids is known. Divide these masses by that volume, and we obtain the density in each case.

Compare your results with those obtained in the previous exercise.

5. Counterpoise a shallow dish and a stopper (with a fibre attached) on one pan, against shot on the other pan of a balance. Now immerse the stopper in a beaker of water supported independently of the balance, and add from a burette a sufficient volume of water to the dish to maintain the counterpoise.

Note that the volume of water added is equal to the volume of the stopper. Prove that this depends upon the fact that the *same number stands approximately for both mass and volume in the case of water.*

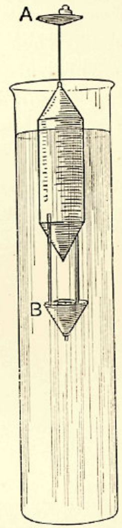


FIG. 122.—A form of hydrometer. The body to be measured is first placed at *A*, and then at *B*.

6. Use a hydrometer[1] (Fig. 122). Weigh a body on the balance. Place it on the top of the hydrometer as it floats in water in a cylinder, and mark on the stem of the hydrometer the level of the water.

Then move the body to the lower pan of the hydrometer, and note the additional mass which must be added *to the upper pan* in order to depress the hydrometer till the original mark on the stem is again at the level of the water.

This additional mass represents the apparent loss when the body is immersed in water, and enables you to calculate the density of the body. The following example will illustrate the process:

Mass of body $= \left\{ \begin{array}{l} \text{20 grams (a 20 gram} \\ \text{weight was used).} \end{array} \right.$

$\left. \begin{array}{l} \text{Additional mass needed to depress} \\ \quad \text{hydrometer to same mark as in-} \\ \quad \text{dicated the level of water when the} \\ \quad \text{body was placed on the top} \end{array} \right\} = 2{\cdot}45 \text{ grams.}$

[1] It may be noted here that hydrometers may be very cheaply obtained for class use, by having the body and other parts made separately of tin-plate, according to pattern, by a local tin-plate worker. These may then be fitted together with wire, and adjusted in the school workshop. Care must be taken that there is no hole left by which the water may leak into the hydrometer.

Volume of water measuring 2·45 grams $= \frac{2 \cdot 45}{\cdot 999} = 2 \cdot 452$ c.c.

Density of body $= \frac{20}{2 \cdot 452} = 8 \cdot 15$, about.

But the use of a balance is not necessary. We can easily find the mass of the body by the hydrometer alone. Place a known mass, larger than the body to be weighed, on the hydrometer at the top, and mark the level of the water on the stem. But if, as is probable, there is a mark already there, ascertain the mass needed to depress the hydrometer to that mark, and then find *what mass added to the body* brings the same mark to the level of the water.

For example of this method :

Mass required to sink hydrometer to given mark	= 22	grams.
Mass required together with body	= 2	"
∴ mass of body	= 20	"

The hydrometer may now be used to find the density of a body lighter than water, and the value of the cap at the bottom, in serving to keep down the light body, will be perceived. A method may also be devised for finding the density of a liquid by means of the hydrometer. Find the mass of the hydrometer itself, place it in water and ascertain the mass needed to float it with its mark at level of water. These masses represent the mass of a certain volume of water. What is that volume? And how do you proceed to find the density of a liquid?

The density of the brass in a 20-gram weight should also be found, when methylated spirit is used in the place of water for floating the hydrometer. The results should be compared.

7. Make a mixture of water and spirit which enables paraffin wax to remain in it at any position in which it may be placed, and prove that the wax and the mixture then have the same density.

PRECAUTIONS TO BE OBSERVED IN MEASURING DENSITY.

184. In order that density may be correctly ascertained, by the process of immersing a body in a liquid, and finding out thereby the volume, it is essential that

the **density of the liquid** should be accurately known. This density may be determined by experiment, or we may refer to the records of the most careful determinations. But in the latter case we must be sure, that the liquid, whether water or another substance, is under the **same physical conditions** when we use it as it was when its density was measured.

In the first place the water, if that liquid be used, must be **pure.** It often contains substances in solution, and we have learnt that the density is affected by matter held in solution. Distilled water is free from solids in solution, but it may contain dissolved air. This may be expelled almost wholly by boiling.

Another condition of the greatest importance is the **temperature** of the water. Tables may be referred to in other books[1] which will give the exact density at various temperatures. To prove the importance of this matter, it is only necessary to show that the counterpoise of a body suspended in water is considerably altered when the water is warmed. At the temperature at which most experiments in the laboratory are peformed, the density of water may be taken to be ·999. When boiling, its density is ·958, and when about to freeze it is practically 1.

It is also necessary to take care that **no bubbles of air** have been carried down with the body. If any are noticed, they should be removed. A frequent error on the part of beginners is to allow the body to touch the side of the vessel containing the water.

The fibre used for suspension should be as light as possible. One of silk is to be preferred, as silk has about the same density as water, and does not absorb water.

[1] Lupton's *Tables*, for example.

In case the body which has to be measured be soluble in water, it must be suspended in a liquid which does not dissolve it.

MORE EXACT MEASUREMENT OF TIME.

185. When we come to attempt a subdivision of time which is finer than that obtained by the use of a clock or watch, we are met by numerous difficulties. It is easy enough to measure length and mass, *quantities which are fixed and constant;* but the essence of time is *change.* It is easy to refer again to the mass or the length measured, to estimate it again; but it is not possible to measure time over again. It is a change which marks a period of time, and a change can only be measured once.

We can, it is true, measure *similar changes* over and over again, and we may assume them to be equal in duration. Once provided with a change which repeats itself uniformly, we can always refer to it. **Thus it is that the pendulum places time on a level with mass and length, in enabling a given quantity of time to be measured repeatedly.** Notice:

1. *A given length may be measured repeatedly, provided the particles of matter which limit that length are not displaced with regard to one another.*

2. *A given mass may be measured repeatedly, provided there is no separation of the parts of that mass sufficient to prevent their total effect being marked on the instrument which is used.* (The necessity for introducing this condition is perceived when we deal with a body which readily loses particles by evaporation. For example, a liquid, such as ether, when exposed in an open vessel, would give different measurements

at different times. We know that the particles are still present in the air of the room, but they cannot be weighed or measured when so dispersed.)

3. *The quantity of time which is limited by two isolated events cannot be compared with a standard quantity of time more than once. It is impossible to return to that quantity and measure it again with a view to greater accuracy.*

186. These three statements express an **important difference** in the nature of the three primary quantities, length, mass, and time. It is true that we assume, and feel secure in assuming, that certain events recur at equal intervals of time. Among such events are *the rotation of the earth; the journey of the earth round the sun; the oscillation of a pendulum within the same arc; the oscillation of a spring, as in a watch, or of an elastic bar, such as the prong of a tuning-fork; and the fall of the same quantity of a liquid under the same conditions through an aperture.*

The time of all these periods has, nevertheless, some elements of uncertainty. We should not feel justified in assuming them to be equal among themselves, but for the fact that *one kind of change may be tested by setting it against several other kinds of change.* The assumption of equality seems reasonable when all the different tests *agree in giving concordant results.*

Being unable therefore to apply to time the test which is so valuable in the measurement of mass and length, namely, **repetition**, we must have recourse to some of **those changes which recur uniformly,** that is, at equal intervals of time. These may be made to repeat themselves indefinitely. **A given pendulum may be made to swing whenever it is required, and so we have a fixed standard** to which we can refer as often as is needed. There are other uniform periods which can be referred to repeatedly; but *irregular* periods can be measured once only.

187. In rendering the measurement of time as accurate as possible, the only course open to us is to use either a method in which an event is repeated, *and also recorded*, at the smallest interval of time, or one in which a uniform displacement takes place in such a way as to be *observable over a finely divided scale.* The vibration of a tuning-fork forms an illustration of the first method; and the movement of the second-hand over the face of a stop-watch is an example of the second method.

A tuning-fork may be fixed, so that a pointer attached to a prong may mark a sheet of paper which is made to move rapidly past it. The line traced will be of the form shown (Fig. 123); and each distance *ab*, *bc*, *cd*, etc., marks an equal interval of time.[1]

In order to compare a given short interval of time with that marked by the successive vibrations of the tuning-fork, the events limiting that interval must be made to mark the sheet of paper at the same time as the tuning-fork. If an object be undergoing a change, and that change can be marked in any way on the sheet of paper by the side of the line showing the movements of the tuning-fork, a comparison of periods

[1] What that interval is may be ascertained either by reference to a pendulum, or by means of an instrument called a syren, which registers the number of vibrations made by it in a second as it gives out the same note as the tuning-fork. Or, again, we may measure the rate at which the sheet moves, and consider the marks made by the fork as equal fractions of a known period of time.

of time is at once possible. Such a comparison is shown in Fig. 123, where the line *bc* marks the interval of time between the two events denoted by *x* and *y*. Each interval on the line *af* marks $\frac{1}{100}$ of a second.[1]

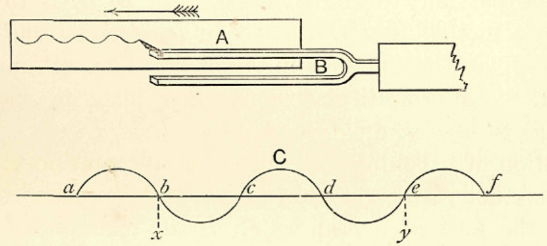


FIG. 123.—A mode of marking small intervals of time. The sheet *A* is caused to move past the vibrating fork *B*, which traces a line *af* as shown at *C*. Two events are denoted by *x* and *y*.

188. The stop-watch affords another method of obtaining smaller quantities of time than are marked by ordinary time-keepers. A stop-watch consists of an ordinary watch with a very large second-hand, which moves during a second over an arc large enough to be subdivided into 5 equal parts. It is fitted with a catch which enables the seconds-hand to be "stopped," and also to be started, by a touch of the finger. A stop-watch may now be examined, and it will be found that the watch marks $\frac{1}{5}$ of a second with accuracy.

It may be noticed that in this watch we have simply *extended the distance to be traversed* by the end of the seconds-hand, and so allowed the distance to be subdivided. This could not be extended indefinitely, for the movement of the hand would then become too

[1] It is clear that very minute fractions can be marked by this means, but it is also clear that the practical work is too difficult for beginners.

quick to be followed by the eye, otherwise we might mark very small fractions of a second. It is necessary that the stop-watch should be started and stopped by pressing a spring, as previously stated. This introduces the chance of error, and is a serious defect in spite of the fact that *the time lost in starting* the watch is probably equal to *that lost in stopping it.*

It may be noted that in both the methods of measuring time which have been described, we have had to measure a length. There is much to be gained by **regarding time as something analogous to length,** and by *representing it as a length* whenever possible.

189. But in addition to the method of extending or adding to the space passed over by the moving body which marks time, a method exhibited both by the stop-watch and the instrument previously described, there is a useful method of measuring very small *differences of time.*

Two pendulums, very nearly equal in length, are supported one behind the other, so that when at rest and viewed through a slit, the one pendulum covers the other, or rather the wire or support of the one covers that of the other. Now if these two pendulums swing in nearly equal periods, and they be started together, the one will slowly gain on the other. They will coincide at the start, pass the slit separately after a time, then in course of time again pass the slit together. **The interval of time between any two coincidences represents the time occupied by the quicker pendulum in performing one more oscillation than is performed by the slower pendulum.**

If the slower one oscillate in seconds, and it be found that 120 seconds elapse before it is caught up again by the quicker pendulum, which started level with it, then the quicker pendulum oscillates 121 times in 120 seconds; and **the difference between the times of oscillation of the two pendulums is $\frac{1}{120}$ of a second.** Very minute differences of time may be detected by means of this method of noting the coincidences, *but there is no opportunity of making use of the difference when measured.* **It cannot be applied as a standard for making other measurements.**

EXERCISES IN MORE EXACT MEASUREMENT OF TIME.

1. Find out the difference in the length of the second, as marked by the watches provided (one watch having had its regulator altered).

2. Find out the difference in the periods of time in which the two given pendulums oscillate, using the method of coincidences.

3. Ascertain the effect on the time of oscillation caused by placing a magnet close underneath the iron ball of the pendulum provided.

4. Measure the rate of your pulse by means of a clock. Holding the thumb or second finger over the pulse, count the beats for several periods of three minutes each. Note if there be any variation in the beats.

5. A and B start on the 1st of August to go round the world. A goes east, B goes west. They travel on an average at the same speed. They keep a careful diary, but when they meet again at the club from which they started, they disagree as to the day of the month. A says it is Nov. 2nd, B says it is Oct 31st. Explain their disagreement.

CHAPTER XI.

SOME SIMPLE CHANGES REQUIRING EXACT MEASUREMENT.

INTRODUCTION.

THE following section contains a variety of observations which may be easily carried out in any laboratory. The majority of experiments have the advantage of not requiring expensive material. And such experiments as have been described serve as examples of a large number of the same kind which will suggest themselves. They have been selected with a view to practice in accuracy of measurement, but in all cases the results observed have an important practical value as leading to generalizations, that is, general statements which may be made of a class. For example, the relation which is observed to exist between the bending of a beam and the load it carries, holds true not only for the beam under observation, but for all beams of the same dimensions and material. At the same time it must be remembered that we are not justified in assuming that what has been observed in one case applies to all others, unless the conditions are known to be exactly the same.

A generalization is a statement which holds true for a class of objects or changes. Frequently a generalization is spoken of as a *law*, but the word should always be used very carefully in this connection. In ordinary language, a law is a regulation which ought to be obeyed. There is a law to prevent stealing, but it is often disobeyed. **But a law of nature cannot be disobeyed,** for it is nothing more than a statement of what does take place. In fact, if it were possible to break a law of nature, then the law would be a false one. It would be false in not including the case to which it ought to apply, and which it is supposed to include. A law of nature shows the observed relation between facts and magnitudes. **It does not state what ought to exist, but what is observed to exist.** Hence it will be perceived that the common error of supposing that laws of nature *explain* facts is disastrous from the point of view of logic and correct reasoning. At the same time we must be very cautious in making general statements which have the force of laws. Innumerable observations must be made, and all cases which present any possibility of exception to the general rule should be carefully tested. Even when certain up to this point, it should be borne in mind that a law is true only so far as is known. Further research may prove it to be incorrect.

The following observations have been written out for the greater part in note-book fashion, in order to serve as illustrations of how records of practical work should be made in the student's note-book. It should always be remembered, in writing out the results of observations, that no number should ever appear without a statement of the kind of quantity of which it is the value, or the event of which it denotes the order.

Numbers without words should never appear in any record, since they always stand for quantities or for the sequence of events. Most of the observations have been made and recorded by students working in our own laboratory. It seemed to me probable that work which had really been carried on would be felt to be the most stimulating and suggestive. In most cases the results have been recorded in the form of a curve drawn on squared paper, and it should be remembered that curves are to be drawn whenever it is possible to do so. Not only is it valuable as a training in one of the most important branches of practical science, but it will be found that the consideration of, and pondering over quantities, which is involved in expressing them by a diagram, will help you very materially to understand and realize what it is that you are recording. I have mentioned in each case the names of my pupils who have carried out the work. Several of them have been purposely selected on the ground that they are *beginners*, as I wished to show that even beginners may treat their work as a subject of private research; in fact I am convinced that all practical work, however elementary, may be regarded by those engaged in it in the light of an important research.

THE STRENGTH OF BEAMS. BENDING BY LOADS.

Wooden laths of rectangular section, with length and breadth of section approximately in the proportion of 3 to 2, may be used. If these are supported at two fixed points, and loads of varying quantity placed on them at their centre, the lath will be bent to different extents. The arrangement shown in fig. 124 may be used. The edges used for support are cast for the purpose, and screwed down on the wooden stands which have been already described. It will be found that the bending varies with the thickness of the beam and the size of the load.

The first exercise will be to find out the best method of placing a beam to support a load, whether to put it flat or edgewise. In the next place you may find out that the bending varies with the dimensions of the beam. Laths may be placed side by side, or on the top of one another, to illustrate the variation of the amount of bending with the dimensions of the beam. The nature of the material used will be found to exercise considerable effect, and steel bars are recommended as examples of another material for experiment.

You may here notice, too, that the position of the load, and also its distribution, affects the result. 500 grams at the centre of the lath, for example, bends the lath more than 5 separate 100 grams placed at even distances along its length. This is apparent without any accurate measurements. It may also be noticed that the bending is less when the ends of the laths are firmly fixed, as may be done by clumps.

Observations should also be made to show that within certain limits the bending is proportional to the load, but that after a certain load has been added the bending begins to increase more rapidly than the load is being increased.

The length of the lath will be found to exercise a very considerable influence on the bending. The law connecting these quantities should be discovered by varying the length of the lath by means of bringing the supporting edges closer together. In this, as in all experiments in this section, the results of observations should be recorded by curves on squared paper.

Further observations must also be made of the effects of bending produced in the lath when one end alone is fixed. The results which are found should be compared with those obtained in the previous exercises. The observations should be made with laths of various dimensions, and with different materials as in the previous cases. Steel bars and more than one kind of wood should be tested, so as to obtain comparative results.

The condition of material which is being bent may be demonstrated very clearly by a large bar of india-rubber on which parallel lines have been ruled. In Fig. 124 we have the bar at rest, and also showing the change made by a load. The compression of the upper portions and the extension of the lower part may be perceived by measurement.

A few illustrations of investigations of this character will now be described as they were carried on by students.

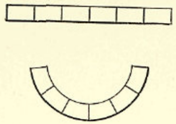


FIG. 124.—A bar of india-rubber on which vertical lines have been ruled to show the changes produced in different parts of the material when it is bent.

EXPERIMENTAL METHOD.

The experiments are best carried out with one or two of the stands with knife-edges which have been described. By means of one stand only, the effect of varying loads on a rod or lath of fixed length can be investigated, and we can also determine the effect of variations in the cross-section, the length, load, and material being the same. With two stands the effect of varying the distance between the knife-edges may be studied.

The middle of the lath should be marked and placed midway between the knife-edges, so that each knife-edge is at the same distance from the end of the lath. The scale pan is then hung to a kind of miniature riding-stirrup, and the stirrup placed midway between the knife-edges. A pin is now fixed by a piece of wax in a vertical position with its point upwards as near to the stirrup as is possible, and on the edge of the lath. A vertical scale is placed as close to the point of the pin as will permit the necessary freedom of motion of the lath. The point of the pin may be brought very close to the scale by inclining the pin from the vertical, but even when this has been done there may be some error introduced, owing to our not viewing the pin in a strictly horizontal direction. Where great accuracy is required it will be better to view the pin and scale from a distance through a telescope.

By the deflection produced by a given load we do not mean the vertical distance between the bottom of the lath when deflected and the straight line joining the knife-edges. The

lath is bent to a certain extent by its own weight and also by the weight of the scale pan and stirrup. Be careful then to measure the deflection corresponding to a given load as the difference in the readings of the position of the pointer against the scale before and after the load is placed in the pan.

In order to use the observations to determine the relation between the deflection and the load, it is better to get rid of accidental errors by recording the results on squared paper. A curve is drawn through the points of observation in the most regular or natural manner possible, so that if it cannot include all the points representing actual observations, yet it leaves as many on the one side of it as upon the other. This curve will more probably represent the true deflection produced by a given load than the curve which aims at including every point, irrespective of irregularities of shape thereby produced.

A SERIES OF OBSERVATIONS ON THE BENDING OF A LATH WITH ITS ENDS ON KNIFE-EDGES.

This experiment was to show the relative bending of a lath according to the loads which were placed upon it.

The lath used was an oak one, its dimensions being: 31·7 in. long, ·55 in. wide, ·24 in. thick. To support it we had two

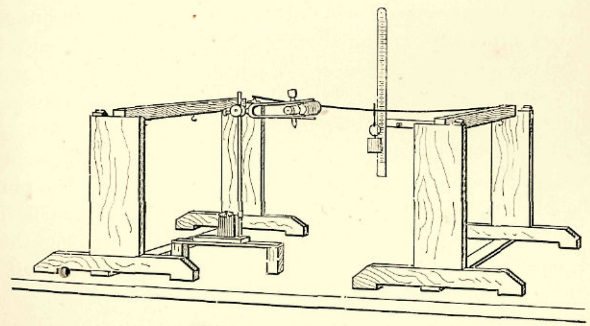


FIG. 125.—Apparatus set up for the observation of the bending produced in a lath by various loads. The lath lies across two iron edges without being otherwise fixed. (From a photograph.)

stands fitted with knife-edges on which the lath rested freely. The knife-edges were placed 29·8 in. apart, and the midway point on the lath being marked, the loads were suspended from that spot; by the side of it a pin was fixed with beeswax. Behind the pin was fixed a scale. A telescope was then fixed up at a convenient distance to read the different positions of the point of the pin.

(i.) Observations when the working length was 29·8 in.[1]

Load Affixed. Lb.	Height of Pin-point. Inches.	Deflection. Inch.
0	7·25	...
$\frac{1}{2}$	7·05	·20
1	6·91	·34
$1\frac{1}{2}$	6·77	·48
2	6·61	·64
$2\frac{1}{2}$	6·39	·86
3	6·22	1·04
$3\frac{1}{2}$	6·07	1·18
4	5·85	1·40
$4\frac{1}{2}$	5·73	1·52
5	5·56	1·69
$5\frac{1}{2}$	5·41	1·84
6	5·27	1·98

(ii.) When the working length was $\frac{2}{3}$ of (i.) or 19·85 in.

Load Affixed. Lb	Height of Pin-point. Inches.	Deflection. Inch.
0	15·32	0
$\frac{1}{2}$	15·26	·06
1	15·22	·10
$1\frac{1}{2}$	15·16	·16
2	15·12	·20
$2\frac{1}{2}$	15·08	·24
3	15·03	·29
$3\frac{1}{2}$	14·98	·34
4	14·94	·38
$4\frac{1}{2}$	14·88	·44
5	14·83	·49
$5\frac{1}{2}$	14·79	·53
6	14·75	·57

[1] Observations made by A. Rixon.

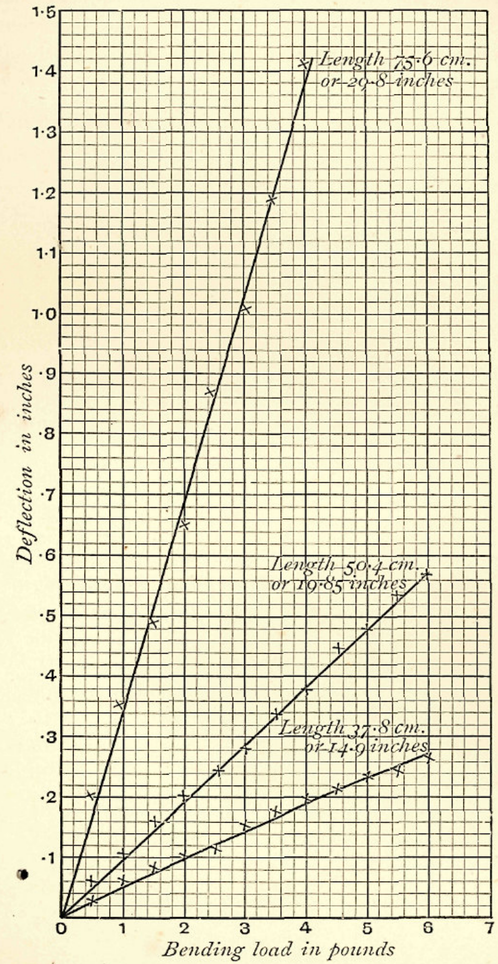


FIG. 126.—Curves representing the bending produced by varying loads on the same lath, which is supported on two edges placed at different distances apart, namely 75·6, 50·4, and 37·8 centimetres, so as to show the relation of the bending to the length of lath as well as to the load.

(iii.) With length 14·9 in. = ½ (i.)

Load Affixed. Lb.	Height of Pin-point. Inches.	Deflection. Inch.
0	15·70	0
½	15·67	·03
1	15·64	·06
1½	15·62	·08
2	15·60	·10
2½	15·58	·12
3	15·55	·15
3½	15·53	·17
4	15·51	·19
4½	15·49	·21
5	15·47	·23
5½	15·45	·25
6	15·43	·27

DEFLECTION OF A ROD WITH BOTH ENDS FIXED.[1]

The loads were placed half way along the rod, and readings taken as described in the last investigation.

Breadth of rod = 1·3 centimetres; thickness = ·55 centimetre.

Free length (1) = 75·6 centimetres.

Loads. Lb.	Reading of Scale.	Deflection. Cm.	Difference.
0	4·50	0	0
½	4·26	·24	·24
1	4·02	·48	·24
1½	3·80	·70	·22
2	3·58	·92	·22
2½	3·35	1·15	·23
3	3·10	1·40	·25
3½	2·85	1·65	·25
4	2·60	1·90	·25
4½	2·32	2·18	·22
5	2·05	2·45	·27

[1] Observations made by C. T. Scott.

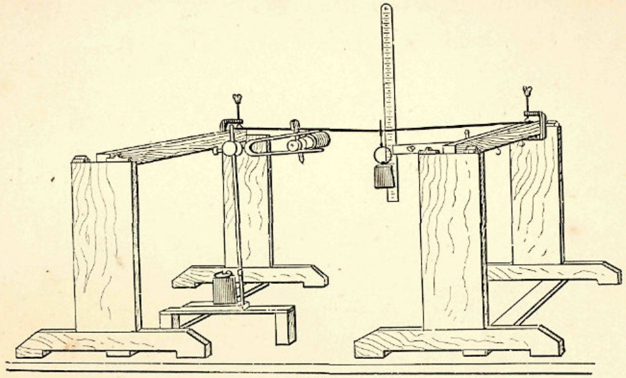


FIG. 127.—The same apparatus as in the last figure, modified for the purpose of observing the bending produced by various loads when the ends of a lath are fixed by clamps. (From a photograph.)

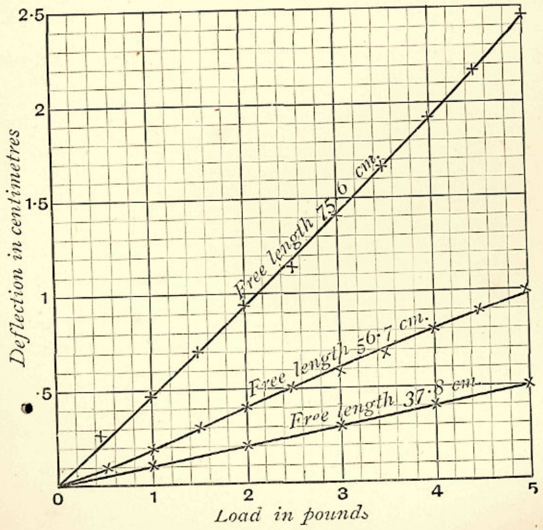


FIG. 128.—Curves showing the variation in the extent of bending of a lath when the ends are firmly fixed by clamps. The deflection is measured in centimetres.

Free length (2) = 56·7.

Loads. Lb.	Reading of Scale.	Deflection. Cm.	Difference.
0	40·85	0	0
$\frac{1}{2}$	40·72	·13	·13
1	40·63	·22	·09
$1\frac{1}{2}$	40·54	·31	·09
2	40·45	·40	·09
$2\frac{1}{2}$	40·34	·51	·11
3	40·25	·60	·09
$3\frac{1}{2}$	40·14	·71	·11
4	40·05	·80	·09
$4\frac{1}{2}$	39·95	·90	·10
5	39·84	1·01	·11

Free length (3) = 37·8.

Loads. Lb.	Reading of Scale.	Deflection. Cm.	Difference.
0	41·7	0	0
1	41·6	·1	·1
2	41·5	·2	·1
3	41·4	·3	·1
4	41·3	·4	·1
5	41·2	·5	·1
6	41·1	·6	·1
7	41·0	·7	·1

DETERMINATION OF RELATION BETWEEN THE BENDING OF A LATH AND ITS LENGTH.[1]

The observations were made with apparatus essentially the same as that shown in Fig. 127, but the lath was supported by clamping at one end only. The pin at the free end of the rule was attached by a piece of wax, so that its point moved parallel to the edge of a small finely divided boxwood scale (containing 60 divisions to the inch), which was fixed in a vertical position immediately behind the point of the pin. The movement of the pin's point over the scale was read by a suitable

[1] Made by C. C. Roberts.

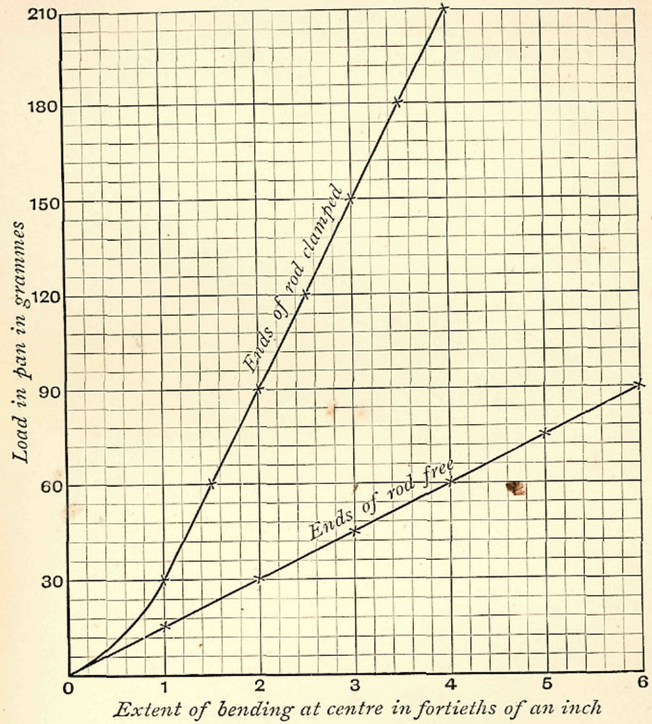


FIG. 129.—Curves showing the deflection produced by varying loads on a steel rectangular bar, both when the ends are free and when the ends are clamped. This diagram, which represents the results of another set of observations, is constructed so that the quantities are not represented on the same co-ordinates as in the previous diagrams.

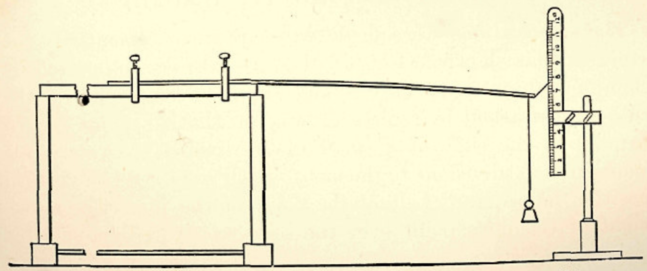


FIG. 130 A.—Apparatus used in observing the bending produced in varying lengths of the same lath by the same load.

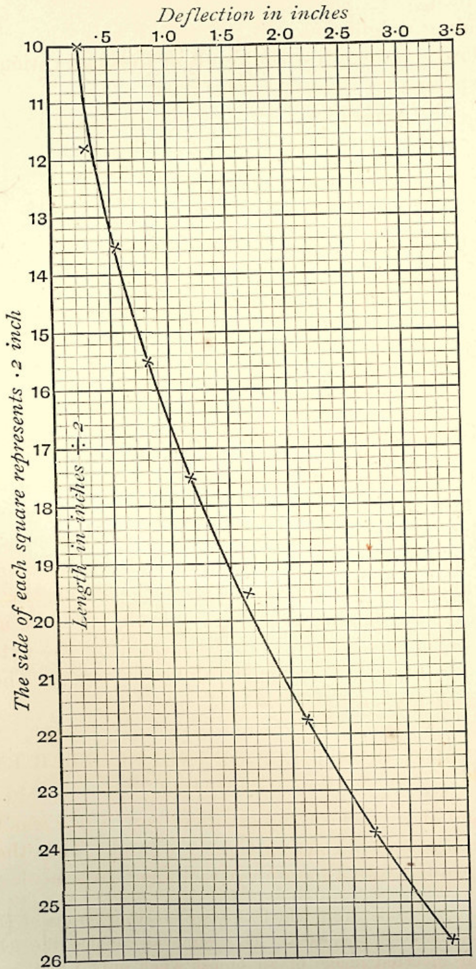


FIG. 130 B.—Curve showing the bending of various lengths of the same lath under the same load (1 lb.). Total length of lath 31·75 in., breadth ·58 in., depth ·28 in.

telescope.[1] With this arrangement the magnitude of a deflection could be read to within a quarter of a scale division, *i.e.*, $\frac{1}{240}$ of an inch.

The bending weight was 1 lb.

The following table gives the result of the observations which are also expressed in the curve, Fig. 130 B:

Length. Inches.	Deflection. Inches.
10	·26
11·81	·33
13·50	·55
15·5	·80
17·5	1·17
19·5	1·65
21·7	2·12
23·7	2·70
25·75	3·34

THE STRETCHING OF BODIES BY LOADS.

The effect of loads upon bodies in extending or stretching them may now be observed. The bending or flexure of most materials by a load is more familiar than the extension, and it is certainly much more easy to produce. If we were to try to produce extension of the wooden laths of the dimensions which have been used for bending, very heavy loads would be needed. Hence we begin our observations by measuring the extension of a piece of india-rubber under different loads. Then we proceed to measure springs.

EXTENSION OF A PIECE OF INDIA-RUBBER BY VARYING LOADS.[2]

To one end of a piece of india-rubber cord a pan was firmly attached by means of fine string tightly wound. The other end was tightly wound into a loop for attachment to a hook. Suc-

[1] An ordinary telescope, from which the two erecting lenses of the eye-piece had been removed, was used. This removal enabled the telescope to give a clear inverted image of an object when only 5 feet from the object glass.

[2] Observations made by R. B. Coare.

T

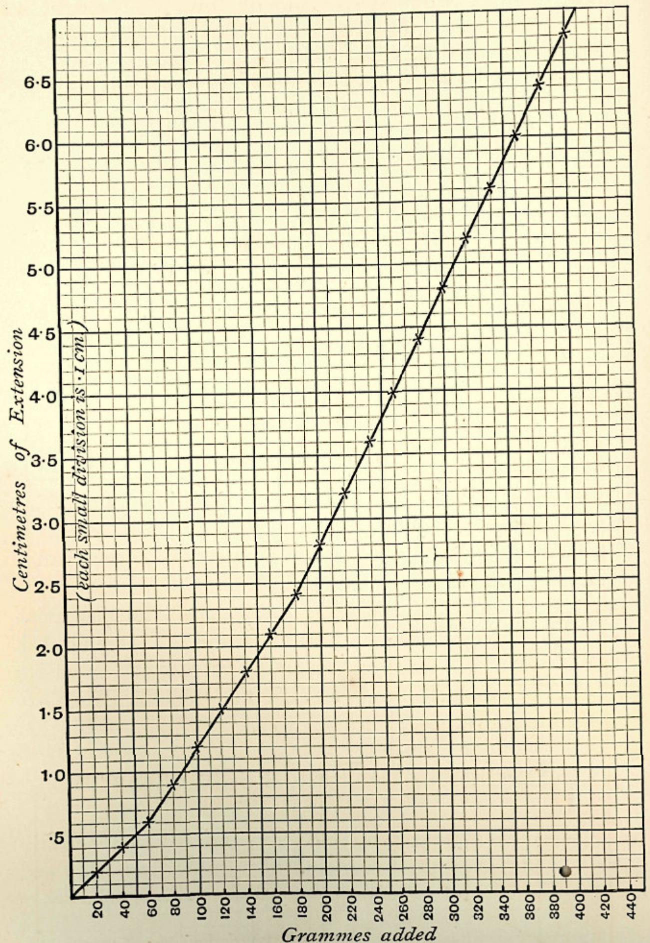


FIG. 131 A.—Curve showing the extension of an india-rubber cord 5 mm. in diameter.

cessive loads of 20 grams were added, and the extension was indicated by a wire pointer moving over an upright scale

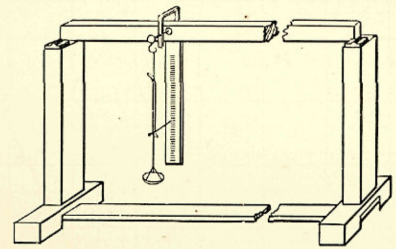


FIG. 131 B.—Apparatus used in observing the extension produced in an india-rubber cord by varying loads.

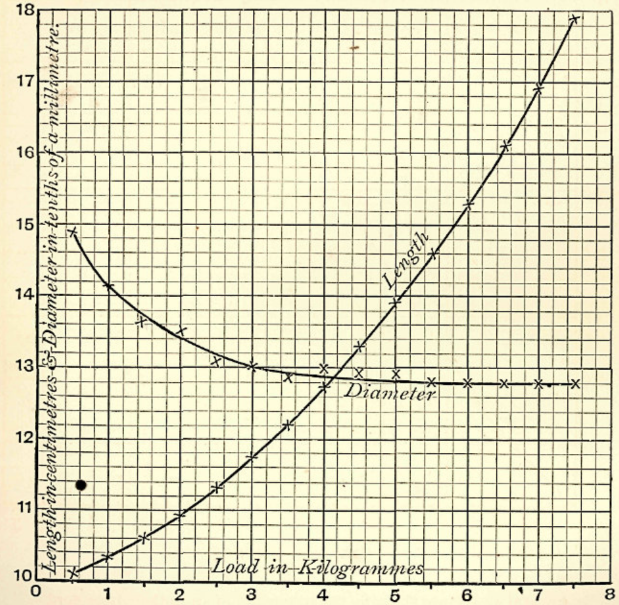


FIG. 132.—Curve showing the extension of another india-rubber cord under varying loads, together with a curve showing the change produced at the same time in the diameter of the cord.

placed behind the cord, and also by the increase of length between two marks made at some distance apart on the cord. A telescope was used for reading the extension.

MEASUREMENT OF THE EXTENSION OF A BRASS SPRING BY A WEIGHT NOT SUFFICIENT TO PERMANENTLY DISTORT IT.[1]

The spring used was made of 115 cm. of brass wire, ·55 mm. in diameter, coiled round a cylinder about ·9 cm. in diameter by rotating the cylinder in a lathe and winding the wire so that each turn was in contact with the preceding one; on carefully drawing out the cylinder, a flat spiral spring with 40 turns and about 8 cm. long was left; the two ends were twisted into loops to hang the spring to a hook and to hold the scale pan in which the weights were placed; the lower loop had the free end of the wire sticking out in a horizontal direction for about 1 cm. so as to form an index or pointer, the position of which could be read on a millimetre scale fixed behind the spring. To avoid any error due to the position of the eye with respect to the pointer and scale, the readings were taken with a telescope arranged for use at short distances.[2] The amount of extension produced was the difference between the position of the pointer when the scale pan alone was attached to the spring, and when the given mass was placed in the pan. After each reading with the different masses in the pan, the position of the pointer when the pan was empty was read, to ascertain whether any permanent extension of the spring had been produced, and this was not found to be the case until 50 grams had been placed in the scale pan. The following readings were taken:

Mass in Pan. Grams.	Amount of Extension. Cm.	Mass in Pan. Grams.	Amount of Extension Cm.
5	·5	30	3·0
10	1·0	35	3·5
15	1·5	40	4·0
20	2·0	45	4·5
25	2·5	50	5·0 to 5·1

[1] Made by T. B. Hornblower.

[2] The apparatus used resembled that shown in Fig. 131 B, with the exception that a telescope was used, as seen in Fig. 127.

At 50 grams the spring began to get fatigued, and at 50·5 there was a permanent extension of 2 cm.

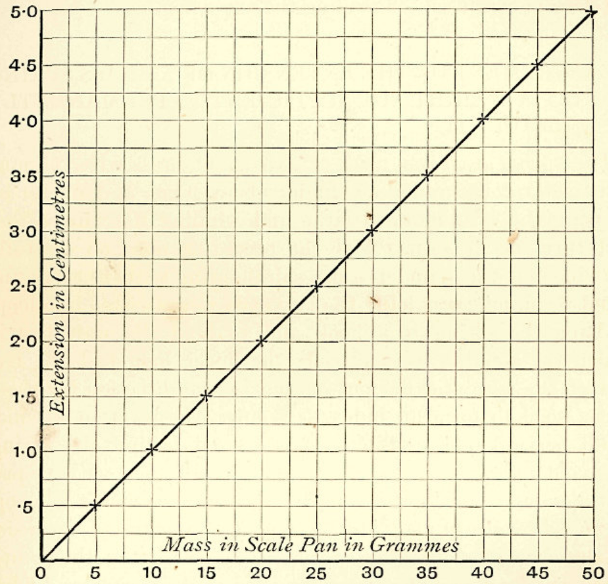


FIG. 133.—Curve showing the relation between the extension of a spiral spring and the load producing that extension.

DETERMINATION OF STRETCHING OF BRASS WIRE.

Distance of support from floor, 455·3 cm. Distance of pin on stretched wire from floor when pan is empty, 126·5 cm.

∴ length of wire = 455·3 − 126·5 = 328·8 cm.

The diameter of the wire obtained by a number of observations with the screw-gauge was 1·15 mm.

Experiment with increasing load.

Stretching Load. Lb.	Reading. Mm.	Difference.	
0	6·7	0	(1)
10	5·9	·8	(2)
20	4·3	2·4	(3)
25	3·5	3·2	(4)
30	2·8	3·9	(5)

$$\left.\begin{array}{l}(1)-(3)=2{\cdot}4\\(2)-(5)=3{\cdot}1\end{array}\right\}\text{mean } 2{\cdot}7 \text{ for } 20 \text{ lbs.} = 1{\cdot}35 \text{ for } 10 \text{ lbs.}$$

(2)-(4)=2·4 „ 15 „ =1·6 „

(1)-(4)=3·2 „ 25 „ =1·28 „

80) 11·1 „ 80 „

·139 mm. = mean for 1 lb.

Experiment with decreasing load.

Stretching Load. Lb.	Reading. Mm.	
40	·6	(1)
35	1·2	(2)
30	2·0	(3)
25	2·7	(4)
20	3·4	(5)
15	4·4	(6)

(4)-(1) = 2·1 for 15 lbs.

(5)-(2) = 2·2 „ 15 „

(6)-(3) = 2·4 „ 15 „

6·7 „ 45 „

and mean for 1 lb. $=\frac{6{\cdot}7}{45}=1{\cdot}49$.

The mean for 1 lb., taking the results obtained with both increasing and decreasing loads, was ·144 mm.

(From the above observations Young's modulus can be calculated.)

MEASUREMENT OF THE THICKNESS OF A BRASS DISC.[1]

METHOD I. (i.) *By screw-gauge (measuring inches).*

This instrument depends upon the movement of a fine screw, the pitch of which is as uniform as possible.

α. The first measurement to be taken is to ascertain the value of the divisions. In this case

	1 large division of the scale represented	0·1	inch,
also	1 small „ „ „	0·02	„
and	1 division of the circular scale „	0·001	„

[1] By J. A. Wheäller.

β. The next observation to be taken is to see if the zero on the scale was coincident with the true zero of the instrument.

In this case they were found to be coincident.

γ. Observations must now be taken in the usual way through each of the eight segments into which the disc has been first of all marked. One of the segments may be marked so that when different instruments are used the same portion of the disc may be always measured.

δ Measurements of the various segments.

Five readings were taken on each of the eight segments.

Segment. No.	Reading. No. 1. Inch.	No. 2. Inch.	No. 3. Inch.	No. 4. Inch.	No. 5. Inch.	Mean. Inch.
1.	·1236	·1235	·1235	·1236	·1234	·1235
2.	·1237	·1232	·1238	·1235	·1234	·1235
3.	·1217	·1219	·1213	·1212	·1217	·1215
4.	·1199	·1200	·1198	·1203	·1199	·1199
5.	·1190	·1201	·1191	·1190	·1190	·1192
6.	·1193	·1200	·1192	·1195	·1195	·1195
7.	·1212	·1215	·1207	·1215	·1210	·1211
8.	·1227	·1226	·1224	·1230	·1227	·1226

$$\left.\begin{array}{l}\text{Mean of 40 observations, giving}\\ \textit{average}\text{ thickness of disc}\end{array}\right\}=\cdot1213 \text{ inch.}$$

Inaccuracies may be due to insufficient care in calibrating the instrument, and also to changes of temperature which cause the material to expand.

In these observations the temperature is supposed to remain constant.

(ii.) *By screw-gauge (measuring centimetres).*

α. As before, 1 large division of the scale represented	0·1	cm.
1 small „ „ „	0·05	„
1 division of the circular „	0·001	„

β. The true zero of the instrument coincided with the zero on the scale.

γ. Observations were taken as before.

Measurements of the various segments.

Segment. No.	Reading. No. 1. Cm.	No. 2. Cm.	No. 3. Cm.	No. 4. Cm.	No. 5. Cm.	Mean. Cm.
1.	·314	·314	·314	·314	·314	·314
2.	·313	·312	·314	·313	·314	·313
3.	·309	·310	·311	·308	·309	·309
4.	·305	·306	·305	·304	·305	·305
5.	·302	·305	·303	·302	·302	·303
6.	·303	·305	·302	·304	·303	·303
7.	·307	·308	·306	·309	·308	·308
8.	·312	·312	·311	·312	·312	·312

Mean of 40 observations, giving *average* thickness of disc } = ·308 cm.

METHOD II. (i.) *By spherometer (measuring centimetres).*

α. 1 large division on scale represented 0·1 cm.
1 small " " " 0·05 "
1 large circular division " 0·001 "

β. The zero of the instrument was at the 10th division, *i.e.*, at 1 cm.

γ. Measurements of the various segments.
Seven readings were taken on each segment.

Segment. No.	Reading. No. 1. Cm.	No. 2. Cm.	No. 3. Cm.	No. 4. Cm.	No. 5. Cm.	No. 6. Cm.	No. 7. Cm.	Mean. Cm.
1.	·687	·687	·688	·697	·688	·687	·688	·687
2.	·688	·688	·689	·689	·688	·689	·689	·689
3.	·689	·689	·690	·690	·690	·691	·690	·690
4.	·691	·692	·691	·692	·691	·691	·691	·691
5.	·692	·692	·692	·692	·691	·691	·692	·692
6.	·691	·691	·691	·691	·691	·691	·691	·691
7.	·690	·690	·691	·691	·690	·690	·690	·690
8.	·689	·690	·690	·689	·689	·689	·689	·689

Mean of 56 observations = ·690.
Therefore *average* thickness of disc = 1·0 − ·690
= 0·310 cm.

Inaccuracies may be due to the same causes as in the case of the observations with the screw gauge.

Temperature was supposed to remain constant.

(ii.) *By spherometer (measuring inches).*

α. 1 division on long scale represented 0·01 inch,
1 " circular " 0·0001 "

β. The true zero of the instrument (*i.e.*, the point at which all the four feet touch the plane) was at 26 long scale divisions, and 7 circular scale divisions.

γ. Measurements of the various segments.
Seven readings were taken on each segment.

Segment. No.	Reading. No. 1. Inch.	No. 2. Inch.	No. 3. Inch.	No. 4. Inch.	No. 5. Inch.	No. 6. Inch.	No. 7. Inch.	Mean. Inch.
1.	·1380	·1386	·1378	·1379	·1380	·1381	·1387	·1381
2.	·1380	·1385	·1380	·1378	·1380	·1386	·1388	·1383
3.	·1393	·1386	·1388	·1390	·1386	·1389	·1390	·1389
4.	·1397	·1394	·1386	·1391	·1389	·1392	·1386	·1390
5.	·1394	·1396	·1389	·1385	·1391	·1391	·1387	·1390
6.	·1397	·1395	·1399	·1398	·1402	·1401	·1399	·1398
7.	·1393	·1391	·1387	·1388	·1390	·1386	·1384	·1388
8.	·1383	·1380	·1388	·1388	·1385	·1384	·1387	·1383

Mean of 56 observations = ·1388.
Therefore *average* thickness of disc = ·2607 − ·1388
= ·1219 inch.

COMPARISON OF SCREW-GAUGES.

As an exercise in accurate measurement the same length may be measured by two different screw-gauges, or other instruments, one of which is graduated in centimetres and the other in inches. The ratio of the length in inches to that in centimetres should be ·3937.

NOTE.—In using a screw-gauge, take care to ascertain the value of the divisions on the scale and screw-head, as gauges by different makers are apt to be differently graduated. Another point to be noticed is the determination of the "index error," or the reading of the gauge when its jaws are just in contact. Lastly all objects should be held with same degree of tightness in the jaws, and that as small a one as possible, or else the screw will be strained.

The following measurements were made with two gauges (by C. C. R.):

$$\text{Length of object A.}\left\{\begin{array}{l}\cdot 3916 \text{ inch}\\ \cdot 99 \text{ cm.}\end{array}\right\}, \text{ Ratio } \cdot 3936.$$

$$\text{Thickness of object B.}\left\{\begin{array}{l}\cdot 1280 \text{ inch}\\ \cdot 325 \text{ cm.}\end{array}\right\}, \text{ Ratio } \cdot 3938.$$

$$\text{Length of object C.}\left\{\begin{array}{l}\cdot 4346 \text{ inch}\\ 1\cdot 104 \text{ cm.}\end{array}\right\}, \text{ Ratio } \cdot 3937.$$

INVESTIGATION OF THE BEHAVIOUR OF A BALANCE UNDER DIFFERENT CONDITIONS.[1]

The object of these experiments was to observe the behaviour of a balance when loaded with an equal mass on each pan and also when unloaded.

To observe the effect of placing on one pan a very small weight, such as 2 or 3 milligrams, when the pans were empty and also when loaded, formed a third investigation.

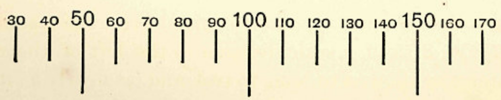


FIG. 134.—The numbers given to the division-marks on the scale in investigating the balance.

The balance selected was one of moderate accuracy, used for weighing in the chemical laboratory to within 5 milligrams.

The first thing to be done was to find as accurately as possible the resting-point of the balance when both pans were empty.

The scale of the balance used was divided by lines as shown in Fig. 134, and, in order to read as near as possible, each division was considered as 10, and the divisions called by the numbers as in the figure.

It was now found that when the readings were large (*i.e.*, near

[1] By J. D. Adams.

170 or 30) the pointer made an angle with the scale division, so that it was very hard to read accurately to what extent the pointer was deflected. This is a defect in construction which could be obviated by dividing the scale in such a manner that all the lines marking scale divisions should be parts of the radii of a circle, of which the centre is the point about which the pointer turns, *i.e.*, a point in a line with the knife-edge of the beam.

Again, the readings could be taken with far more certainty if the pointer were thinner. While the pointer is swinging the covering of a considerable portion of the scale by its point seriously interferes with the accuracy of the reading.

The following readings were made, the numbers (1), (2), etc., indicating the order in which they were taken (in this case they were given by consecutive swings):

Left.	Right.
(1) 40	(2) 149
(3) 50	(4) 140
(5) 60	(6) 131
(7) 68	Mean = 140
Mean = 54·5	

$$\therefore \text{ resting-point} = \frac{140 + 54\cdot 5}{2} = 97\cdot 25.$$

This means that when the pointer comes to rest the reading will be: 97·25, *i.e.*, 2·75 of a scale division to the left of the centre. The pointer was allowed to come to rest, and (as nearly as it could be read) it seemed to stand somewhat to the left of this position.

The above series of readings were repeated several times, till 5 positions of rest were calculated to be:

96·125, 95·25, 95·9, 95·9, 95·125.

The mean of all these readings gives, as the resting-point with both pans empty, 95·925. The first result was probably inaccurate through want of practice, since the subsequent results were fairly concordant.

A.

Observations were now taken to find to what extent the resting-point was moved when ·005 gram was placed on the right-hand pan.

The readings were taken in the same manner as before, the results being:

Left.	Right.
(1) 30	(2) 132
(3) 38	(4) 128
(5) 40	(6) 120
(7) 45	Mean = 126·66
Mean = 38·25	

$$\text{From these, the resting-point} = \frac{126{\cdot}66 + 38{\cdot}25}{2} = 82{\cdot}455.$$

Again,

Left.	Right.
(1) 30	(2) 130
(3) 36	(4) 124
(5) 40	(6) 120
(7) 45	Mean = 128
Mean = 37·75	

$$\therefore \text{ resting-point} = \frac{128 + 37{\cdot}75}{2} = 82{\cdot}875.$$

Two other readings were taken in the same way, giving respectively resting-point = 81·55 and 81·83.

The mean of all these readings gives, as the resting point with ·005 grams on right-hand pan, 82·1775.

This gives as the deviation, produced by ·005 gram when balance is unloaded, 13·7475, *i.e.*, between 1 and $1\frac{1}{2}$ scale divisions, and this was seen to be the case when the balance came to rest.

B.

The next thing was to load the pans equally with 5 grams, and see if the resting-point had altered from the unloaded position.

It was now found that the balance became steadier, and that two consecutive swings on either side were so near together that no difference could be read. In order to overcome this difficulty the readings were taken in the following manner.

The readings (1) and (2) were taken and then one swing each side was omitted. Afterwards (5) and (6) were taken, and then

two omitted, and so on, taking care to leave one out on each side between (1) and (10). The results were:

Left.	Right.
(1) 32	(2) 153
(5) 38	(6) 149
(9) 42	(10) 145
(13) 46	Mean = 149
Mean = 39·5	

$$\therefore \text{ resting-point} = \frac{149 + 39{\cdot}5}{2} = 94{\cdot}25.$$

This reading is near enough to show that the resting-point is not appreciably changed by equal loads of 5 grams on each pan.

C.

All the above readings were taken on Friday. On Saturday it was found that the sensibility of the balance had considerably altered. This may be due to several causes.

(1) Dust may have been on the knife-edges which a shake may have displaced by Saturday.

(2) The conditions of temperature and dryness of the air may be, and probably are, very different in the evening of one day and the morning of the next; and although this may not produce any great effect on the resting-point, it might possibly alter the sensibility of the balance.

It was found that the resting-point with no load had not altered to any readable extent, so ·005 gram was placed on the right-hand pan and the resting-point taken again.

The readings were:

Left.	Right.
(1) 40	(2) 110
(3) 47	(4) 108
(5) 50	(6) 102
(7) 51	Mean = 106·6
Mean = 47	

$$\therefore \text{ resting-point} = \frac{106{\cdot}6 + 47}{2} = 76{\cdot}8.$$

Two other sets of readings gave 77·65 and 77·1.

$$\therefore \text{ Mean resting-point} = 77{\cdot}183.$$

Subtracting this from the mean resting-point when unloaded, 18·742 is obtained, a number which divided by 5 gives the *sensibility* of the balance with nothing on the pans.

$\therefore$ Sensibility = 3·7484.

By sensibility we mean the difference of position of the pointer produced by the addition of ·001 *gram to one pan.*

D.

The next operation was to test the balance with 5 grams on each pan, and the readings were :

Left.	Right.
(1) 34	(2) 160
(3) 37	(4) 153
(5) 40	(6) 148
(7) 42	Mean = 154·3
Mean = 37·5	

$$\therefore \text{ resting-point} = \frac{37{\cdot}5 + 154{\cdot}3}{2} = 95{\cdot}9.$$

This shows that the load of 5 grams does not alter the resting-point.

Now 5 grams were placed on left-hand pan, and 5·005 on right-hand.

The readings then were :

Left.	Right.	Left.	Right.	Left.	Right.
(1) 30	(2) 118	(1) 30	(2) 112	(1) 36	(2) 111
(3) 34	(4) 111	(3) 32	(4) 110	(3) 38	(4) 110
(5) 39	(6) 107	(5) 35	(6) 109	(5) 40	(6) 109
(7) 42		(7) 38		(7) 41	
Mean = 36·25	112	33·75	110·3	38·75	110
Resting-point = 74·125.		Resting-point = 72·025.		Resting-point = 74·375.	

Mean resting-point = 73·508.

Subtracting this from 95·925, and dividing by 5, gives us the sensibility of the balance with 5 grams on each pan.

$\therefore$ Sensibility = 4·483.

Thus we see that by putting 5 grams on each pan the balance becomes steadier, and the sensibility greater, than with no load.

E.

The balance was then tested with 10 grams on each pan, and the results were :

Left.	Right.	Left.	Right.	Left.	Right.
(1) 30	(2) 161	(1) 59	(2) 135	(1) 72	(2) 120
(3) 38	(4) 159	(3) 61	(4) 131	(3) 74	(4) 119
(5) 40	(6) 153	(5) 62	(6) 130	(5) 76	(6) 118
(7) 45		(7) 65		(7) 78	
Mean = 38·25	157·6	61·75	132	75	119
Resting-point = 97·925.		Resting-point = 96·875.		Resting-point = 97.	

Two other readings gave 95·625 and 95·275, and these give a mean of 96·193. As this is only the mean of 5 observations, and the original resting-point 95·95 was the result of a great many more observations, the latter is taken as being more correct than the former, though the difference between the two is quite unreadable when the pointer is at rest.

Now 10 grams were placed on the left pan, and 10·005 on the right.

The readings then were :

Left.	Right.	Left.	Right.	Left.	Right.
(1) 30	(2) 126	(1) 30	(2) 121	(1) 48	(2) 108
(3) 31	(4) 122	(3) 32	(4) 120	(3) 50	(4) 104
(5) 33	(6) 120	(5) 38	(6) 116	(5) 51	(6) 100
(7) 37		(7) 40		(7) 53	
Mean = 32·75	122·6	35	119	50·5	104
Resting-point = 77·675.		Resting-point = 77.		Resting-point = 77·25.	

Two other readings gave 76·250, 76·3.

The mean of these is 76·739.

This gives as the sensibility of the balance with 10 grams on each pan 3·8322.

Thus the balance is less sensitive with 10 grams on each pan than with 5 grams on each pan.

F.

The load was then increased to 20 grams on each pan, and the resting-point again taken.

The readings were :

	Left.	Right.	Left.	Right.	Left.	Right.
	(1) 30	(2) 159	(1) 48	(2) 143	(1) 55	(2) 140
	(3) 34	(4) 153	(3) 49	(4) 141	(3) 60	(4) 134
	(5) 39	(6) 150	(5) 50	(6) 140	(5) 62	(6) 130
	(7) 41		(7) 52		(7) 65	
Mean =	36	154	49·75	141·3	60·5	134·6
	Resting-point =95.		Resting-point =95·525.		Resting-point =97·75.	

Mean resting-point = 96·091.

This shows that the resting-point is not altered to any readable extent by a load of 20 grams on each pan.

20·005 grams were now placed on the right-hand pan, and 20 grams on left-hand pan.

The results were :

	Left.	Right.	Left.	Right.	Left.	Right.
	(1) 30	(2) 130	(1) 30	(2) 129	(1) 47	(2) 109
	(3) 32	(4) 124	(3) 33	(4) 122	(3) 51	(4) 108
	(5) 38	(6) 120	(5) 38	(6) 120	(5) 53	(6) 104
	(7) 40		(7) 40		(7) 58	
Mean =	35	124·6	35·25	127	52·75	107
	Resting-point =79·8.		Resting-point =81·125.		Resting-point =79·875.	

Mean resting-point = 80·26.

Subtracting this from 95·925 and dividing by 5, we get 3·133 for the sensibility with 20 grams on each pan.

Thus the sensibility of the balance diminishes with the increase of load after 5 grams.

G.

50 grams were now placed on each pan and the readings taken as before.

	Left.	Right.	Left.	Right.	Left.	Right.
	(1) 30	(2) 145	(1) 40	(2) 136	(1) 45	(2) 131
	(3) 42	(4) 132	(3) 50	(4) 126	(3) 51	(4) 126
	(5) 53	(6) 122	(5) 60	(6) 118	(5) 59	(6) 121
	(7) 61		(7) 69		(7) 64	
Mean =	46·5	133	54·75	126·6	54·75	125·6
	Resting-point =89·75.		Resting-point =90·675.		Resting-point =90·175.	

Mean resting-point = 90·2.

This difference from the previously observed resting-points is probably due to some inequality in the weights. The one 50 gram-weight may not be exactly equal to the sum of the smaller weights making up the other 50 grams.

50·005 grams were now placed in right-hand pan and 50 grams in left-hand pan. The readings then were :

	Left.	Right.	Left.	Right.
	(1) 28	(2) 123	(1) 30	(2) 120
	(3) 30	(4) 119	(3) 33	(4) 116
	(5) 35	(6) 113	(5) 38	(6) 111
	(7) 40		(7) 41	
Mean =	33·25	118·5	35·5	115·6
	Resting-point = 75·875.		Resting-point = 75·55.	

	Left.	Right.	Left.	Right.
	(1) 31	(2) 119	(1) 55	(2) 98
	(3) 38	(4) 112	(3) 58	(4) 96
	(5) 40	(6) 110	(5) 59	(6) 94
	(7) 43		(7) 60	
Mean =	38	113·6	58	94
	Resting-point = 75·8.		Resting-point = 76.	

Mean resting-point = 75·806.

This gives 2·878 as the sensibility with 50 grams on each pan, thus showing that the sensibility still decreases with the increase of load.

H.

Further observations were then made to see whether a load of 100 grams altered the sensibility to any great extent. The results were as follows :

Left.	Right.	Left.	Right.	Left.	Right.
(1) 33	(2) 161	(1) 37	(2) 152	(1) 70	(2) 122
(3) 49	(4) 140	(3) 42	(4) 146	(3) 72	(4) 120
(5) 61	(6) 130	(5) 50	(6) 140	(5) 75	(6) 117
(7) 70		(7) 55		(7) 78	
Mean = 53·25	143·6	46	146	73·75	119·6
Resting-point 98·425.		Resting-point = 96.		Resting-point = 96·675.	

Mean resting-point with 100 grams on each pan = 96·7.

100·005 grams were now placed in the right-hand pan, while 100 grams remained in the left-hand pan, and the readings again taken :

Left.	Right.	Left.	Right.	Left.	Right.
(1) 41	(2) 123	(1) 50	(2) 119	(1) 46	(2) 123
(3) 43	(4) 111	(3) 60	(4) 109	(3) 55	(4) 111
(5) 65	(6) 100	(5) 70	(6) 100	(5) 66	(6) 101
(7) 76		(7) 78		(7) 76	
Mean = 58·75	111·3	64·75	109·3	59·5	111·6
Resting-point = 85·025.		Resting-point = 87·025.		Resting-point = 85·55.	

Mean of all these = 85·866.

Subtracting this from 96·7 and dividing by 5, we get 2·166 for the sensibility of the balance with 100 grams on each pan.

The next thing to be done is to find at what point between 0 gram and 10 grams the sensibility is a maximum.

I.

In order to find the load with which the balance has maximum sensibility, it was tested first with 4 grams on each pan and then with 4·005 on right-hand pan.

These observations gave a sensibility of 3·56.

Evidently, then, the sensibility was still increasing, so readings were taken with 6 grams on each pan, and the resting-point was found to be almost exactly equal to the mean (95·925) resting-point taken in the first observations.

·005 gram was then added to the right-hand pan, and the readings were :

Left.	Right.	Left.	Right.	Left.	Right.
(1) 42	(2) 96	(1) 40	(2) 101	(1) 49	(2) 92
(3) 62	(4) 78	(3) 51	(4) 90	(3) 68	(4)
(5) 64	(6)	(5) 63	(6) 80	(5)	(6)
(7)		(7) 71		(7)	
Mean = 56	87	56·25	90·3	58·5	92
Resting-point = 71·5.		Resting-point = 73·275.		Resting-point = 75·25.	

Mean resting-point = 73·291.

This gives a sensibility of 4·558.

This is slightly more sensitive than with 5 grams on the pans. Evidently, then, the maximum lies between 5 grams and 10 grams.

It may be noticed that in the last observations the readings which were taken decreased very rapidly, and in the second set the readings approached so near to the mean resting-point that the balance was almost at rest when the last were taken.

K.

The above observations were taken on Monday, and on Tuesday morning between 7 and 8 o'clock the sensibility was found to be different.

The balance was tried with 8 grams on each pan, and the resting-point taken :

Left.	Right.	Left.	Right.	Left.	Right.
(1) 34	(2) 169	(1) 41	(2) 160	(1) 51	(2) 151
(3) 45	(4) 159	(3) 51	(4) 150	(3) 61	(4) 143
(5) 54	(6) 149	(5) 60	(6) 142	(5) 69	(6) 138
(7) 64		(7) 69		(7) 74	
Mean = 49·25	159	55·25	144	63·75	144

Resting-point = 104·125. Resting-point = 99·625. Resting-point = 103·875.

Mean resting-point = 102·541.

Now ·005 gram were added to right-hand pan, and readings were :

Left.	Right.	Left.	Right.	Left.	Right.
(1) 50	(2) 121	(1) 40	(2) 130	(1) 58	(2) 115
(3) 52	(4) 119	(3) 43	(4) 128	(3) 59	(4) 113
(5) 56	(6) 115	(5) 49	(6) 123	(5) 60	(6) 111
(7) 59		(7) 51		(7) 61	
Mean = 54·25	118·3	45·75	127	59·5	113

Resting-point = 86·275. Resting-point = 86·375. Resting-point = 86·25.

Mean resting-point = 86·3.

Subtracting this from 102·541 and dividing by 5, we get 5·248 for the sensibility of the balance with 8 grams on each pan.

L.

On returning to the experiment between 9 and 10 A.M. certain apparently extraordinary results induced me to take the sensibility with 8 grams again, and this time the results were :

Left.	Right.
(1) 36	(2) 160
(3) 40	(4) 150
(5) 46	(6) 148
(7) 50	
Mean = 43	152·6

Resting-point = 97·8.

·005 gram was then added to the right-hand pan, and the readings taken again :

Left.	Right.
(1) 40	(2) 115
(3) 43	(4) 112
(5) 45	(6) 110
(7) 48	
Mean = 44	112·3

Resting-point = 78·1.

Subtracting this from 97·8 and dividing by 5 we get 3·98 as the sensibility. 10 grams on each pan was then tried again, and the sensibility found to be 2·627, thus differing very considerably from the value of the previous day.

These results may be put in this form :

(1) Sensibility with 8 grams on each pan	= 3·98	} 10 a.m. Tuesday.
(2) " " 10 " "	= 2·627	}
(3) " " 8 " "	= 5·248	7 a.m. Tuesday.
(4) " " 10 " "	= 3·832	3 p.m. Monday.

EXPERIMENTS WITH AN OERTLING BALANCE.

A.

To find the resting-point with nothing on the balance :

Left.	Right.	Left.	Right.	Left.	Right.
(1) 61	(2) 146	(1) 77	(2) 132	(1) 55	(2) 150
(3) 64	(4) 143	(3) 79	(4) 131	(3) 60	(4) 147
(5) 68	(6) 140	(5) 80	(6) 129	(5) 63	(6) 145
(7) 70		(7) 82		(7) 66	
Mean = 65·75	143	79·5	130·3	61	147·3

Resting-point = 104·375. Resting-point = 104·9. Resting-point = 104·15.

Mean resting-point with no load = 104·475.

·003 gram was now placed on the right-hand pan :

Left.	Right.	Left.	Right.
(1) 50	(2) 108	(1) 51	(2) 108
(3) 54	(4) 104	(3) 56	(4) 103
(5) 59	(6) 100	(5) 58	(6) 100
(7) 61		(7) 60	
Mean = 56	104	56·15	103·6

Resting-point = 80. Resting-point = 79·925.

Left.	Right.
(1) 63	(2) 97
(3) 65	(4) 95
(5) 66	(6) 93
(7) 68	
Mean = 65·5	95

Resting-point = 80·25.

Left.	Right.
(1) 50	(2) 109
(3) 54	(4) 105
(5) 58	(6) 101
(7) 61	
55·75	105

Resting-point = 80·375.

Mean resting-point with ·003 gram on right-hand pan = 80·1

From this we obtain for sensibility with no load, 8·112.

B.

An equal load of 5 grams was now placed on each pan, and the resting-point again taken :

Left.	Right.
(1) 75	(2) 134
(3) 79	(4) 130
(5) 81	(6) 128
(7) 84	
Mean = 79·75	130·6

Left.	Right.
(1) 58	(2) 150
(3) 60	(4) 147
(5) 62	(6) 144
(7) 65	
61·25	147

Resting-point = 105·175. Resting-point = 104·175.

From these readings we obtain a mean of 104·675, which is near enough to the mean resting-point with no load to show that the resting-point has not altered to any readable extent by the addition of 5 grams to each pan.

·003 gram was now added to the right-hand pan, and the results were :

Left.	Right.
(1) 50	(2) 117
(3) 53	(4) 114
(5) 56	(6) 111
(7) 58	
Mean = 54·25	114

Resting-point = 84·125.

Left.	Right.
(1) 62	(2) 106
(3) 64	(4) 104
(5) 66	(6) 102
(7) 68	
65	104

Resting-point = 84·5.

Left.	Right.
(1) 50	(2) 119
(3) 53	(4) 115
(5) 56	(6) 112
(7) 58	
54·25	115·3

Resting-point = 84·775.

The mean of the readings is 84·466, and by subtracting this from 104·475 and dividing by 3, we get 6·669 as the sensibility of the balance, with a load of 5 grams on each pan.

C.

10 grams were now placed on each pan, and the readings were :

Left.	Right.
(1) 51	(2) 146
(3) 55	(4) 142
(5) 58	(6) 140
(7) 60	
Mean = 56	142·6

Resting-point = 99·3.

Left.	Right.
(1) 72	(2) 124
(3) 74	(4) 122
(5) 76	(6) 121
(7) 77	
74·75	122·3

Resting-point = 98·525.

Left.	Right.
(1) 50	(2) 146
(3) 53	(4) 143
(5) 56	(6) 141
(7) 60	
54·75	143·3

Resting-point = 99·025.

Mean resting-point with 10 grams on each pan = 98·95.

·003 gram was now added to right-hand pan, and readings again taken :

Left.	Right.
(1) 54	(2) 107
(3) 56	(4) 105
(5) 58	(6) 103
(7) 59	
Mean = 56·75	105

Left.	Right.
(1) 50	(2) 110
(3) 53	(4) 108
(5) 55	(6) 106
(7) 57	
53·75	108

Resting-point = 80·875. Resting-point = 80·875.

Mean resting-point (with 10 grams on left, and 10·003 on right-hand pan) = 80·875.

Subtracting this from 98·95 and dividing by 3, we obtain 6·025 as the sensibility of the balance with 10 grams on each pan.

From these values for the sensibility, with 0, 5, and 10 grams on each pan, it seems probable that the addition of a load to the balance so alters the relative positions of the parts of the system as to bring into play a new action of those parts.

D.

The balance was now tested with 1 gram on each pan, and the readings obtained were :

Resting-point = 99·5, 98·875, 97·675, 99·55, 99·125, 99·875, and the mean of all these is 99·1.

·003 gram was then added to the right-hand pan, and the readings were :

79·375 and 79·525, giving a mean of 79·45.

Subtracting this from 99·875 and dividing by 3, we obtain 6·55 for the sensibility, with 1 gram on each pan.

This value induced me to take the sensibility with 5 grams on each pan again, and the mean of five observations of the resting-point was 85·3, and from this we obtain as the sensibility 6·391.

The results may be tabulated as follows :

Load (in grms.).	Sensibility.
0	8·112
1	6·55
5	6·53
10	6·025

INVESTIGATION OF THE EFFECTS PRODUCED BY A MASS WHEN SUPPORTED IN A CERTAIN MANNER.[1]

A given mass is supported, by means of a rigid rod or lath, on two bodies capable of showing how much they are affected. For example, the balances shown in the illustration (Fig. 135) are both altered at the same moment when the lath is laid across them. They are still more changed when the mass is laid on the lath. The effects of the mass at the ends of the rod are registered by the balances. The effects are observed to vary according to the position of the body producing them.

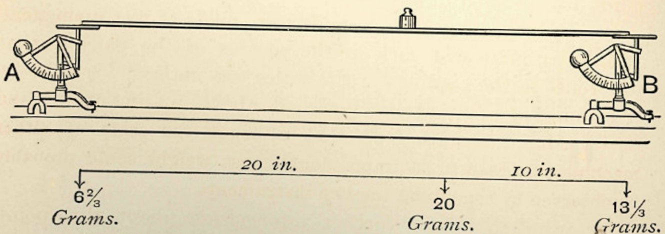


FIG. 135.—The apparatus used in testing the effects of a mass supported in a certain manner.

The object of the following experiments was to test the suitability of the apparatus described, and the degree of accuracy which could be obtained by its careful use, the end

[1] By V. H. Jackson.

in view being to ascertain if the effect of a mass varies inversely with its distance from the point where that effect is produced.

The two pans of the instruments supported a light wooden lath 30 inches in length, which was subdivided into fractions of an inch. Its weight was first determined by placing it on the pans of each machine in turn and was found to be **45 grams.**

It was next placed in position as shown in the figure, and the following readings of the scales were observed :

Left-hand scale, 23 grams ; Right-hand, 22 grams ;
Total, 45 grams.

As it was thought that if the lath was supported upon the edges of the pans the weight might be inaccurate owing to the increased friction of the supports, further readings were taken when the lath rested fully upon both pans.

Left-hand scale, 22½ grams ; Right-hand, 22½ grams ;
Total 45 grams.

so that no appreciable inaccuracy was introduced.

The reason for placing the extremities of the lath upon the edges of the pans was to know the exact point where contact was made and where the effect was produced. Otherwise owing to the shape of the pan it was found impossible to have this point always fixed in position.

In some experiments the ends were made to rest upon small triangular prisms of cork in the centres of the pans, but no difference in the readings of the scales was noticed.

From the observations recorded above, an important and useful fact was gathered,—**that although the scale of either machine only read to 50 grams, double that weight could probably be observed by combining the two instruments.**

In order to obtain further information about this point, different weights were placed upon the middle of the lath, while that was resting on the pans.

The readings observed were as follows :

(1) When no weights were resting upon the lath, the readings of the scales were 23 grams on the left and 22 grams on the right throughout.

(2) With 10 grams on the bar.

Left-hand scale reading, 28 grams; increase 5 grams.
Right-hand „ 27 „ „ 5 „
Total increase, 10 grams.

(3) With 20 grams.

Left-hand scale reading, 33 grams; increase 10 grams.
Right-hand „ $32\frac{1}{2}$ „ „ $10\frac{1}{2}$ „
Total increase, $20\frac{1}{2}$ grams. Error, $\frac{1}{2}$ gram, or $2\frac{1}{2}$ per cent.

(4) With 40 grams.

Left-hand scale reading, 43 grams; increase 20 grams.
Right-hand „ 43 „ „ 21 „
Total increase, 41 grams. Error, 1 gram, or $2\frac{1}{2}$ per cent.

(5) With 50 grams.

Left-hand scale reading, $48\frac{1}{2}$ grams; increase $25\frac{1}{2}$ grams.
Right-hand „ $48\frac{1}{2}$ „ „ $26\frac{1}{2}$ „
Total increase, 52 grams. Error, 2 grams, or 4 per cent.

It was observed that when the weights were removed the scales did not read as before, **until they were made to swing freely for a few moments.** After obtaining some puzzling results it was also found necessary to do the same on adding the weights. In fact, the serious error with the 50 gram-weight seemed to be partly due to the limit of the scale being reached, and hence the movable portion was unable to swing freely.

The probable cause of this behaviour was that the friction between the pans and the wood was slightly called into play as the pans altered in position, especially as for larger weights the inaccuracy was more marked.

Hence when the weight was at the centre of the lath, it was shared by the two machines with considerable accuracy, as the average error was only 3 per cent.

The next step necessary was to take one of the weights and place it upon different points on the lath, to see whether the total always remained constant. The 20 gram-weight was selected as being most suitable.

The following results show that this weight always appeared to weigh $20\frac{1}{2}$ grams:

(1) When no weights were resting upon the bar, the scale readings were always **23 on the left and 22 on the right.**

(2) When it was placed at half way, *i.e.*, 15 inches from either end, the increase on the left was 10 grams.
„ „ right „ $10\frac{1}{2}$ „
Total increase, $20\frac{1}{2}$ grams.

(3) At 10 inches from the left.
Increase on left was $13\frac{1}{2}$ grams.
„ right „ 7 „
Total increase, $20\frac{1}{2}$ grams.

(4) At 12 inches from the left.
Increase on left was $12\frac{1}{2}$ grams.
„ right „ 8 „
Total increase, $20\frac{1}{2}$ grams.

(5) At 5 inches from the left.
Increase on left was 17 grams.
„ right „ $3\frac{1}{2}$ „
Total increase, $20\frac{1}{2}$ grams.

To avoid as far as possible accidental errors, the readings were also taken when the weight was placed at the same distance from the right, and they were found **to correspond exactly.** It was also observed that the constant error of half a gram was **not due to any inaccuracy in the weight itself.**

The above measurements also gave the necessary information about the original problem, namely, to prove that the effect of a mass is inversely proportional to its distance away from the point where the effect is felt. In the first case, when the distances of the weight from the supports were 10 and 20 inches respectively, the weights acting upon the supports were $13\frac{1}{2}$ grams and 7 grams.

Then **$13\frac{1}{2} \times 10$ should have been equal to 20×7,**
or 135 „ „ „ 140.

So that the error was rather less than 4 per cent.

(2) When the distances were 12 and 18 inches, the weights were $12\frac{1}{2}$ and 8 grams respectively.

Then **$12 \times 12\frac{1}{2}$ should have been equal to 18×8,**
or 150 „ „ „ 144.

the error being about 4 per cent.

(3) When the distances were 5 and 25 inches, the weights were 17 and $3\frac{1}{2}$ grams.

Then 5×17 **should have been equal to** $25 \times 3\frac{1}{2}$,

or 85 „ „ „ $87\frac{1}{2}$.

The error was therefore about 3 per cent.

These results showed that with a weight of 20 grams an accuracy to within 4 per cent. could be obtained.

Further experiments showed that the mean amount of error with a weight of 10 grams was about 7 per cent. The cause of this greater error was that a small inaccuracy in the reading, owing to the comparative roughness of the graduations on the scales, was more important with a weight of 10 grams than the same inaccuracy when a larger weight was used.

Several imperfections in the method employed were discovered, some of which could have been easily avoided. In the first place it would have been more convenient and accurate to have **suspended the weights in some kind of cage** attached to any point on the lath by a wire loop. This was due to two reasons.

The first was because no weight larger than 20 grams could be used, as two weights side by side were obviously inadmissible, and the 50 gram weight when near either end was too heavy for the scale at that end.

The second reason was because it was impossible to prevent the weight resting on the bar from touching at more points than at the centre of its base, a fact which might introduce some inaccuracy when the weight was not at the middle of the bar.

Another imperfection, and one which apparently could not be avoided, was that the **bar did not always remain quite horizontal,** because one pan was depressed more than the other when more weight acted upon it. Hence more of the "weight" of the bar would be felt upon that pan. As, however, the difference in level never amounted to more than $\frac{1}{4}$ inch, the increase in weight due to this cause was probably unimportant.

A further inconvenience was the following. It was found to be quite possible to make an error of $\frac{1}{4}$ gram in the reading of the balances. **This error had to be multiplied by a large number** representing the distance of the weight from the end, so that in consequence it was considerably increased.

EXPERIMENT ON THE EFFECTS PRODUCED BY A MASS SUPPORTED ON A BAR.[1]

The following experiment was performed with a straight steel bar weighing $\frac{1}{2}$ lb., with a ring at each end, and a sliding-piece of brass with a hook, to which the mass used could be attached. The remaining apparatus consisted of one of the long stands previously described, and two spring balances graduated to $\frac{1}{4}$ of a lb., and weighing up to 5 lb. The apparatus was arranged as shown in Fig. 136.

The following observations were taken :

Distance of Sliding Mass from No. 1 Spring Balance. Centimetres.	Reading of No. 2. Ounces.	Reading of No. 1. Ounces.
35	20	20
33	19	21
31	18	22
29	17	23
27	16	24
25	15	25
23	14	26
21	13	27
19	12	28
17	11	29
15	10	30
13	9	31
11	8	32
9	7	33
7	6	34
5	5	35
3	4	36
1	3	37

In plotting the readings, half the weight of the bar, *i.e.*, 4 oz., was subtracted from each of the readings of the spring-balances. The curve is intended to represent the readings of the two balances, and the distances of the load from both ends of the bar.

[1] By G. H. Colt.

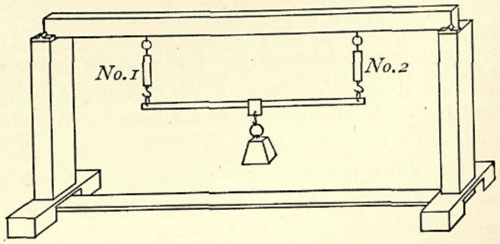


FIG. 136.—The apparatus used in testing the effects produced by a mass supported on a bar.

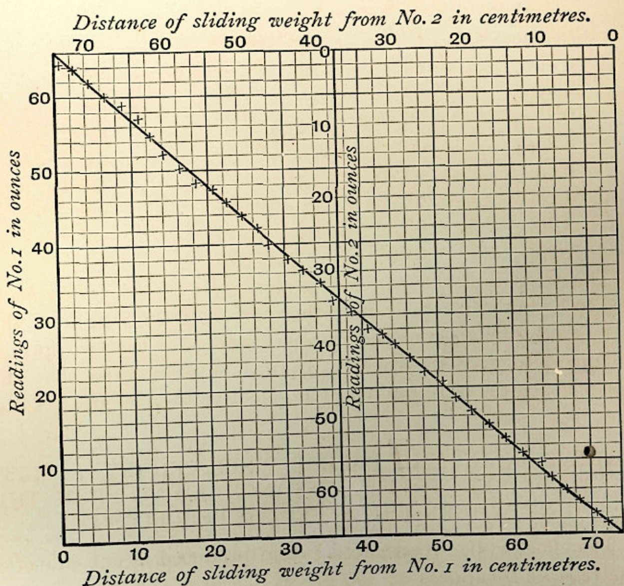


FIG. 137.—Curve representing the observations of the effects produced at the ends of a bar when it supports a load.

ANOTHER SERIES OF OBSERVATIONS OF THE SAME EFFECTS.

Distance of Sliding Mass from No. 1 Spring-balance. Centimetres.	Readings of No. 1 Spring-balance in Ounces after Subtracting the Mass of Bar. Ounces.	Distance of Sliding Mass from No. 2 Spring-balance. Centimetres.	Readings of No. 2 Spring-balance in Ounces after Subtracting the Mass of Bar. Ounces.
36·25	32·5	36·25	32·5
34·25	34·5	38·25	31
32·25	36·25	40·25	28·5
30·25	38	42·25	27·5
28·25	40	44·25	26
26·25	41·5	46·25	24
24·25	43·25	48·25	22
22·25	44·75	50·25	20·25
20·25	47·5	52·25	18·25
18·25	48·5	54·25	16·5
16·25	50·5	56·25	14·5
14·25	52·5	58·25	12·5
12·25	54·5	60·25	11
10·25	56·25	62·25	9·5
8·25	58·25	64·25	7·5
6·25	59·75	66·25	5·5
4·25	61·75	68·25	4
2·25	63·5	70·25	2
1·25	64·5	71·25	1

The weight of the bar, *minus the weight of the sliding hook*, has to be taken into account; therefore half the weight of the bar is subtracted from the various readings of the two spring-balances, to give the correct result. It was noted that the spring-balances were far too little sensitive to give good results.

VARIATION IN THE EFFECT PRODUCED BY A MASS ACCORDING TO ITS DISTANCE AWAY FROM THE POINT WHERE THE EFFECT IS PERCEIVED.[1]

The effect to be investigated is that produced at one end of a metre scale by a mass of 100 grams placed at various points on

[1] Made by T. B. Hornblower.

the metre scale. The effect is made visible by attaching one end of the scale to one arm of a balance, while the other end is supported at the same level. The general arrangement is shown in Fig. 138 A.

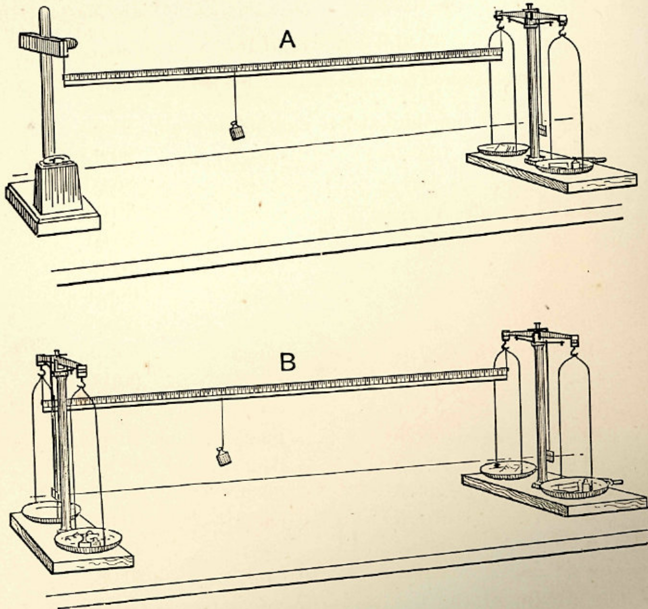


FIG. 138.—The apparatus used in testing accurately the effects produced by a load supported on a bar. *A* represents the arrangement used for obtaining the figures given in the table; while *B* represents an arrangement which gave such accurate results that the effects were inversely proportional to the distances of the load from each balance. (From photographs.)

Two small pins are fixed in the ends of the scale to admit of its suspension by pieces of cotton. The mass of 100 grams is also suspended by a piece of cotton, but before the mass is attached to the cotton the whole is carefully counterpoised by objects placed in the other pan of the balance. The mass is now attached, and the counterpoises required as it is moved from 5 up to 95 centimetres, at intervals of 5 centimetres, are noted. The results are given in the following table.

NOTE.—This is perhaps one of the best examples which can be set to a student for the purpose of giving him an opportunity of discovering a law of nature. There are plenty of laws of nature, but few which can be practically demonstrated with such ease and exactness.

TABLE OF RESULTS.

Amount Required to Counterpoise a 100-Gram Weight.

					Grams.
At	5	cm.	from the balance end	of the scale,	94·99
,,	10	,,	,,	,,	90·00
,,	15	,,	,,	,,	85·03
,,	20	,,	,,	,,	80·03
,,	25	,,	,,	,,	75·04
,,	30	,,	,,	,,	70·05
,,	35	,,	,,	,,	65·04
,,	40	,,	,,	,,	60·05
,,	45	,,	,,	,,	55·11
,,	50	,,	,,	,,	50·11
,,	55	,,	,,	,,	45·10
,,	60	,,	,,	,,	40·06
[1] ,,	60	,,	,,	,,	40·03
,,	65	,,	,,	,,	35·07
,,	70	,,	,,	,,	30·07
,,	75	,,	,,	,,	25·06
,,	80	,,	,,	,,	20·08
,,	85	,,	,,	,,	15·08
,,	90	,,	,,	,,	10·09
,,	95	,,	,,	,,	5·15

PRECAUTION AND EXTENSION OF THE EXPERIMENT.

When taking the observations, notice each time if the pieces of cotton holding the scale up are close against it, and also if the load hangs from the division mark from which it is supposed to hang.

Further observations were made by means of the arrangement shown in B. of Fig. 138. Here both ends of the scale were

[1] N.B.—Owing to the experiment being taken down at this point, it had to be reset and one different drawing-pin used.

supported by balances, and the effect of the load at each end of the scale was measured by a balance in each case. The results were found to be almost exactly what the law of the subject states to be true, namely, that the effect is inversely proportional to the distance of the load away from the point where its effect is perceived. In no case did the observations vary more than 2 in 1000 from the theoretical[1] numbers.

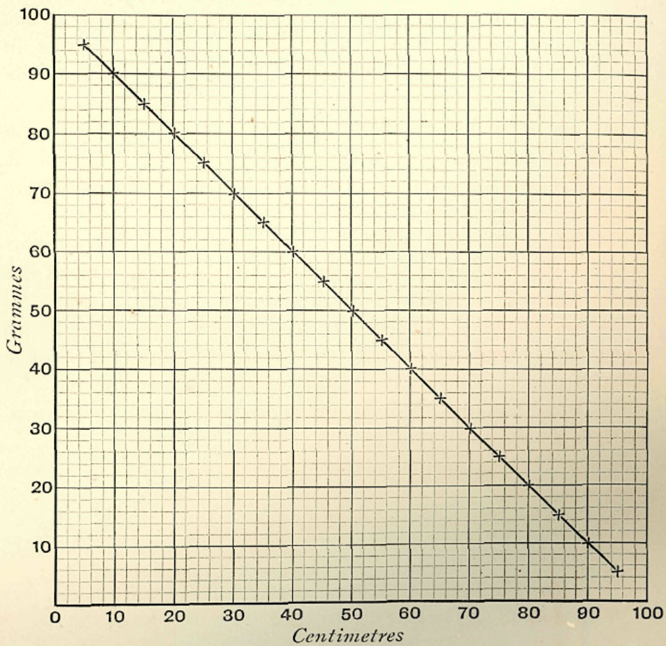


FIG. 139.—Curve showing the variation in the effect of a mass with distance.

[1]The word theoretical is frequently used in conjunction with such words as quantity or value, implying that these values or quantities are such as would conform with the law of nature expressed in the theory in question.

TORSION OF A WIRE.[1]

The amount of twisting or torsion of different points on a wire was determined by the aid of pieces of paper fixed to the wire at nearly equal intervals. These were arranged so that when the wire was at rest they all pointed in the same direction. It was observed that at the centre of each swing they all pointed in the same direction for a moment. This seemed to show that the amount of swing was in some way proportional to the distance of the pointer from the suspension of the wire. The wire was of steel and No. 16, B. W. G.[2] It was 15 feet in length, and kept tightly stretched by a heavy mass at the bottom. The top was firmly fixed so as not to turn.

Experimental difficulties prevented direct comparison of the velocity of two pointers at the middle of their swing, so that it was found necessary to compare the total angles of torsion of the different pointers during a complete swing.

Two observers were needed, one to observe the amount of swing of the lowest pointer, while the other observed that of the higher pointers in succession.

The lowest pointer was of wire, and rotated over a large fixed circle, and the movements of the others were determined by a movable circle. Both of these were graduated into degrees.

The usual amount of swing given to the lowest pointer was nearly a whole revolution.

Several observations of each comparison were taken to ensure greater accuracy.

The results obtained were as follows :

			Lowest Pointer.	Number of Observations.
Top pointer,		$37\frac{5}{7}°$	321°	7
2nd	,,	74°	303°	9
3rd	,,	$92\frac{2}{3}°$	255°	9
4th	,,	$154\frac{1}{6}°$	321°	6
5th	,,	179°	293°	5
6th	,,	$235\frac{1}{4}°$	321°	5
7th	,,	$289\frac{1}{3}°$	335°	6
			Mean = 307°.	

[1] Observed by V. H. Jackson and E. R. Clarke.
[2] Birmingham Wire Gauge.

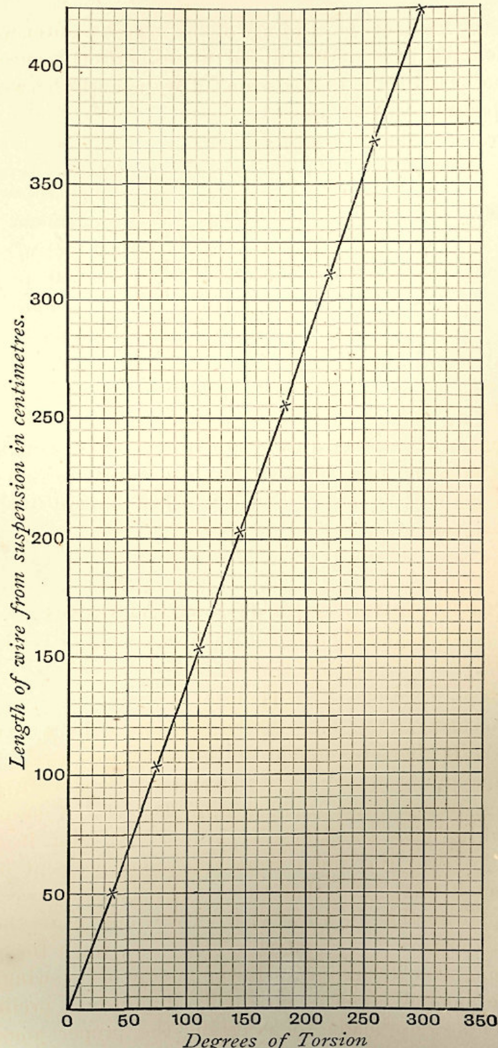


FIG. 140.—Curve showing how the angle, through which a long wire is twisted, varies at different positions along the length of the wire.

Since the mean of the readings of the lowest pointer was 307°, the other readings were reduced in proportion to a constant reading of 300° on the lowest scale. The distances between each pair of pointers was also found.

The result is shown by the table :

			Distance between each Pair.
Torsion shown by	top pointer,	35·25°	
			52 cm.
" "	2nd "	73·26°	
			50 "
" "	3rd "	109·1°	
			50 "
" "	4th "	144·1°	
			55 "
" "	5th "	183·6°	
			53 "
" "	6th "	219·8°	
			58 "
" "	7th "	259·1°	
			57 "
" "	lowest "	300°	

The curve which was drawn to express these results, being almost a straight line, showed that equal distances in length from the point of suspension added an equal amount to the total angle of torsion.

TO ASCERTAIN WHETHER THE ROTATIONS OF A BODY WHEN SUPPORTED BY A WIRE ARE STRICTLY ISOCHRONOUS,[1] WHATEVER THE EXTENT OF SWING MAY BE.[2]

An iron sphere weighing 36 lbs., supported by a steel wire of 16 B. W. G. diameter, and 15 ft. in length, was used.

The amount of torsion was determined by a wire pointer travelling over a circle divided into degrees. A stop-watch was used to give the times of swing. At first it seemed simplest to read the times from the commencement of each swing, but it was found that there was a pause between each change of direction in the twist. The times were therefore taken at the period of greatest velocity, *i.e.*, when the pointer was passing over its zero point in the middle of its swing.

As the amount of swing gradually diminished, preliminary experiments were made to ascertain the proper amount of

[1] Isochronous—of equal lengths of time.

[2] Observations made by V. H. Jackson.

torsion to be given to the wire at the beginning, in order to obtain the required mean amount of swing.

The following results were obtained :

(*a*) When the amount of vibration was 90°.

Time of 25 complete oscillations—

1st observation, 4 minutes $55\frac{3}{5}$ seconds.

2nd „ 4 „ 56 „

Mean, 4 minutes $55\frac{4}{5}$ seconds.

∴ time of 1 complete oscillation = 11·835 seconds.

(*b*) When the pointer vibrated through 360°.

Time of 25 complete oscillations—

1st observation, 4 minutes $56\frac{1}{5}$ seconds.

2nd „ 4 „ $56\frac{3}{5}$ „

Mean, 4 minutes $56\frac{2}{5}$ seconds.

∴ time of 1 complete oscillation = 11·856 seconds.

(*c*) When the average amount of swing was 720°.

Time of 25 complete oscillations—

1st observation, 4 minutes $57\frac{1}{5}$ seconds.

2nd „ 4 „ $57\frac{1}{5}$ „

∴ time of 1 complete oscillation = 11·888 seconds.

(*d*) When the pointer travelled over $3\frac{1}{5}$ complete circles.

Time of 25 vibrations—

1st observation, 4 minutes $57\frac{4}{5}$ seconds.

2nd „ 4 „ $57\frac{4}{5}$ „

∴ time of 1 vibration = 11·915 seconds.

The following table shows the results more clearly :

Amount of Swing. Circle.	Time of Swing. Seconds.	Difference. Second.
$\frac{1}{4}$	11·835	...
1	11·856	+ ·021
2	11·886	+ ·032
$3\frac{1}{5}$	11·915	+ ·027

This shows that the increased amount of vibration causes a corresponding, though very slight increase in the period of each swing. *Whereas it will be seen, that for swings of the same amplitude, the time is as closely identical as can be observed.*

MEASUREMENT OF THE AMOUNT OF CONTRACTION ON MIXING EQUAL VOLUMES OF WATER AND ALCOHOL.

1. Deliver 50 c.c. of alcohol into a tared[1] beaker from a burette.
2. Weigh the alcohol in the beaker.
3. Add 50 c.c. of water from the same burette, after rinsing it, to the alcohol, and allow the mixture to cool.
4. Weigh the mixture of water and alcohol.
5. Empty the same burette, rinse it with some of the mixture, and then fill it from the rest.
6. Deliver 50 c.c. of the mixture into a tared beaker and weigh it.

Mass of alcohol = 41·15 grams.
Total mass of mixture = 90·95 „
Mass of 50 c.c. of mixture = 46·45 „

Before the alcohol and water are mixed they jointly measure 90·95 grams, and occupy 100 c.c. of volume.

When mixed they weigh the same, but occupy a different volume, as is shown in operation 6, for 46·45 grams were found to occupy 50 c.c., whereas $\frac{90·95}{2}$, or 45·475 grams, should occupy 50 c.c. had there been no contraction. That is, 46·45 – 45·475, or ·975 gram, more matter is now packed away in 50 c.c., and consequently 2 × ·975, or 1·95 grams, in 100 c.c.

The question now remains, what volume would these 1·95 grams have filled had there been no contraction?

It was found by weighing, that 90·95 grams of alcohol and water, before being mixed, filled 100 c.c. (50 c.c. water and 50 c.c. alcohol).

∴ 1 gram would fill $\frac{100}{90·95}$ or 1·099 c.c., about,

and 1·95 would fill 1·099 × 1·95 or 2·143 c.c.

That is, when 50 *c.c. of water are mixed with* 50 *c.c. of alcohol, a contraction of* 2·143 *c.c. takes place.*

Note that precautions were taken to measure the bodies under the same conditions, as far as possible.

Also, that the volumes have been measured by means of identically the same marks. It would alter the accuracy

[1] Counterpoised.

possibly, to measure between different graduations of the burette on the several occasions. If we are sure that the temperature is the same throughout, we are quite sure that we are dealing with exactly the same volumes, for they are independent of any *inaccuracy in graduation of the burette.*

EXPERIMENTS WITH PENDULUM.[1]

The following experiments were conducted with a view to ascertaining whether, and in what way, the period of vibration of a pendulum is affected by the distance through which the pendulum moves :

A Kater's pendulum was first taken, and the knife-edges were rested on two horizontal steel plane surfaces, which were fixed on a support in front of a clock, in order to permit comparison between the two pendulums. Great care was taken that the knife-edges should be in no way displaced from their original position during the course of the experiment. By this means the friction at the knife-edges was maintained constant.

The pendulum was entirely made of hard brass, of density 8·2 ; the coefficient of linear expansion being ·000019. The distance, carefully measured, between the two knife-edges was found to be 75·3 cm. The weight of pendulum was 6·5 lbs. The smaller weight of pendulum was down. The atmospheric pressure was 75·5 cm. The width of the bar of the pendulum was 2·4 cm. The temperature was found to be 16° C. on each of the days occupied in observation, and consequently no alteration occurred in the conditions of the experiment on account of change in the dimensions of the pendulum.

A metre scale was placed horizontally close behind the bar of the pendulum. By means of a telescope some feet away, the resting-place of the pendulum was read on the scale behind. This method reduces the error of reading to a minimum. To the corrected reading half the thickness of the bar was added in order to find the position of the centre of the bar. A 56 lb. weight was placed on a board at a distance of about 3 ft. from the pendulum, and about half-way up it. A long string was attached to the weight, and at the end of it a loop of cotton was tied. After the position at the vertical had been

[1] Made by F. L. Golla.

read, the loop of cotton was attached to one of the knife-edges projecting from the lower weight, and the weight was moved along the board away from the pendulum, until the string had displaced the pendulum through a certain angle from the vertical. The position of the middle of the pendulum was then read on the scale, and its linear distance from the vertical reading was measured. This distance represents the tangent of the angle through which the pendulum had been raised.

A light was now applied to the cotton, which rapidly burnt, and thus allowed the pendulum to swing. The rate of vibration of the pendulum was then determined by the method of coincidences, with the aid of the pendulum of the clock behind.

Owing to the fact that the amplitude of vibration gradually diminishes in proportion to the time it has been swinging, and that at a certain time the pendulum would be vibrating under precisely the same conditions which would occur if the pendulum had been started from a certain smaller angle ; the transits and all the observations were not allowed to extend over a longer period than ten minutes.

The results obtained by the experiment are here given, in order that the remarks on them may be easier to follow :

First position, 10 cm. from vertical. Tangent = ·1177. Angle, 6° 42′.
Mean time of one vibration = ·800 second (from 10 coincidences).

Second position, 20 cm. from vertical. Tangent = ·2353. Angle, 13° 12′.
Mean time of one vibration = ·800 second (from 10 coincidences).

Third position, 28 cm. from vertical. Tangent = ·32941. Angle, 18° 12′.
Mean time of one vibration = ·800 second (from 10 coincidences).

Fourth position, 34 cm. from vertical. Tangent = ·4. Angle, 21° 48′.
Mean time of one vibration = ·800 second (from 10 observations with stop-watch).

Fifth position, 40 cm. from vertical. Tangent = ·4705. Angle, 25° 12′.
Mean time, ·8058 second (from 10 observations with stop-watch).

Sixth position, 42 cm. from vertical. Tangent = ·4964. Angle, 26° 24′.
Mean time, ·80522 second (from 10 observations with stop-watch).

Seventh position, 45 cm. from vertical. Tangent = ·5294. Angle, 27° 54′.
Mean time, ·80809 second (from 10 observations with stop-watch).

Eighth position, 50 cm. from vertical. Tangent = ·5882. Angle, 30° 30′.
Mean time, ·8319 second (from 10 observations with stop-watch).

Owing to the rapidity with which the pendulum began to move after the third position, it was found impossible to carry on the observations with any degree of accuracy by the method of coincidences, and consequently a stop-watch had to be used. In order to ascertain the accuracy of this method, results obtained from positions one and two were compared with those already obtained by method of coincidences, and no difference having been found it was deemed safe to use this method for positions four, five, and six. At the fifth position, which is precisely where the increase of time begins, in spite of every precaution the knife-edges were observed to display a tendency to slip along the planes, and out of eleven readings two had to be rejected owing to the knife-edges having been perceptibly seen to slip. In the sixth position, out of ten readings one was rejected on account of slipping. In the seventh position, out of fourteen readings three were rejected. In the eighth position, out of twelve readings four were rejected.

These slippings were indicated by the knife-edges moving from a mark on the plane surfaces, but it was thought probable that the knife-edge might have slipped on the forward swing and have returned to its former position by the backward swing on other occasions without being observed.

The experiment was repeated with a simple pendulum having a ball of iron weighing 1 lb. 5 oz., suspended by a fine brass

thread which had been previously well stretched. The observations were taken, as before, by observing the coincidences of the pendulum with the seconds-pendulum of the clock at different positions from the vertical. The length of the pendulum was 1 m.

Observations, 9th April, 1894, Tonbridge. Temp. 16° C.

First position.

Reading on scale = 11·5 cm. from vertical.
Tangent of angle = ·14375.
Angle = 8° 12′.

Time between two coincidences:

$$\underbrace{13\tfrac{2}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2}{5}}\text{ secs.}$$

Mean time = $13\frac{2}{5}$ secs.
Time of one vibration = ·93056 sec.

Second position.

Reading on scale = 20 cm. from vertical.
Tangent of angle = ·25.
Angle = 14° 6′, about.

Time between two coincidences:

$$\underbrace{13\tfrac{2}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2}{5}}\text{ secs.}$$

Mean time = $13\frac{2}{5}$ secs.
Time of one vibration = ·93056 sec.

Third position.

Reading on scale = 35 cm. from vertical.
Tangent of angle = ·4375.
Angle = 23° 42′, about.

Time between two coincidences:

$$\underbrace{13\tfrac{2}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2}{5}}\text{ secs.}$$

Mean time = $13\frac{2}{5}$ secs.
Time of one vibration = ·93056 sec.

Fourth position.

Reading on scale = 48 cm. from vertical.
Tangent of angle = ·6.
Angle = 31°, about.

Time between two coincidences :

$$\underbrace{13\tfrac{2}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2\cdot25}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2\cdot25}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2}{5}}\ \text{secs.}$$

Mean time $=13\frac{2\cdot07}{5}$ secs.

Time of one vibration $=\cdot93062$ sec.

Fifth position.

Reading on scale $=64$ cm. from vertical.
Tangent of angle $=\cdot8$.
Angle $=38°\ 42'$, about.

Time between two coincidences :

$$\underbrace{13\tfrac{2\cdot25}{5},\ 13\tfrac{2\cdot5}{5},\ 13\tfrac{2\cdot25}{5},\ 13\tfrac{2\cdot25}{5},\ 13\tfrac{2}{5},\ 13\tfrac{2\cdot15}{5},\ 13\tfrac{2\cdot25}{5},\ 13\tfrac{2}{5}}\ \text{secs.}$$

Mean time $=13\frac{2\cdot2}{5}$ secs.

Time of one vibration $=\cdot93075$ sec.

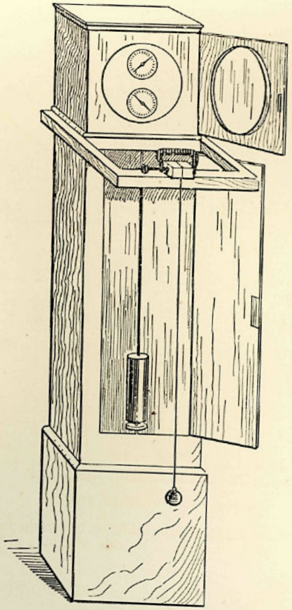


FIG. 141.—The arrangement of a pendulum supported in front of the pendulum of a standard clock, so as to enable their periods to be compared.

In these observations the possible errors of the previous experiment, due to slipping of the knife-edges, were avoided, as the wire was clamped between two blocks of wood. The stop-watch used was only graduated to fifths of a second. The fractional numbers given were judged by the eye with the help of a magnifying glass.

OBSERVATIONS OF THE FRICTION BETWEEN TWO PIECES OF WOOD.[1]

A board of pitch pine 2·83 inches in width was used, and a piece of the same wood, 7·87 inches long and of the same width as the board, was placed on it. The friction observed was that

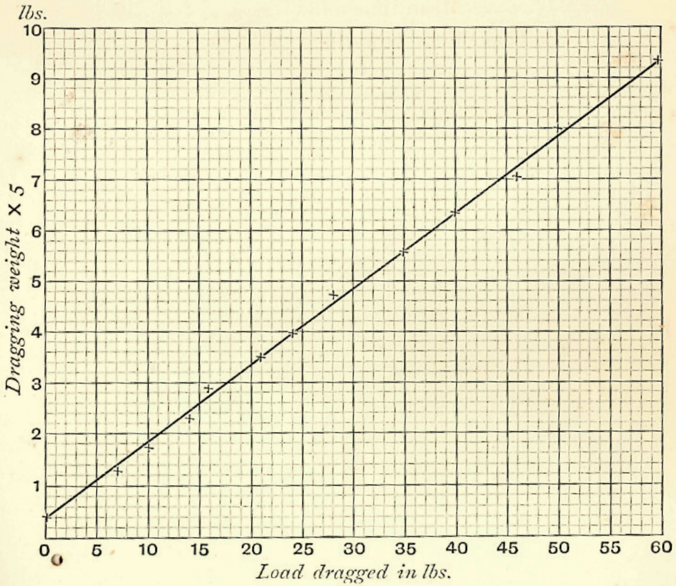


FIG. 142.—Curve showing how the friction between two boards varies with the load carried by one of the boards.

which exists between the two pieces of wood when the one is dragged over the other. Various loads were placed on the

[1] Made by O. Ievers and E. Sells.

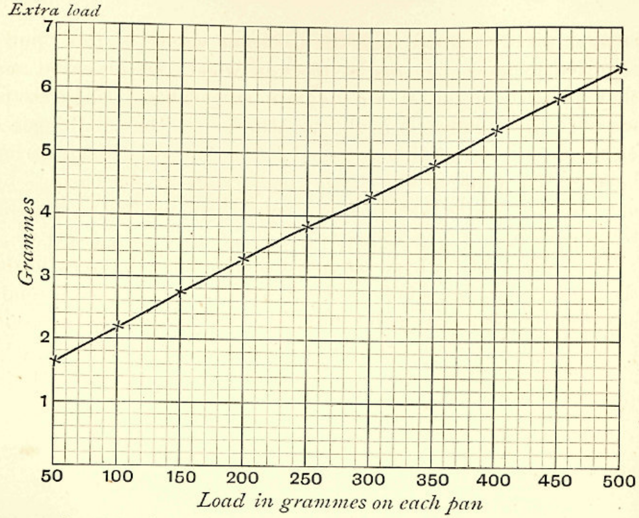


FIG. 143.—Curve showing observations of the friction of a pulley under varying loads. (By J. D. Adams.)

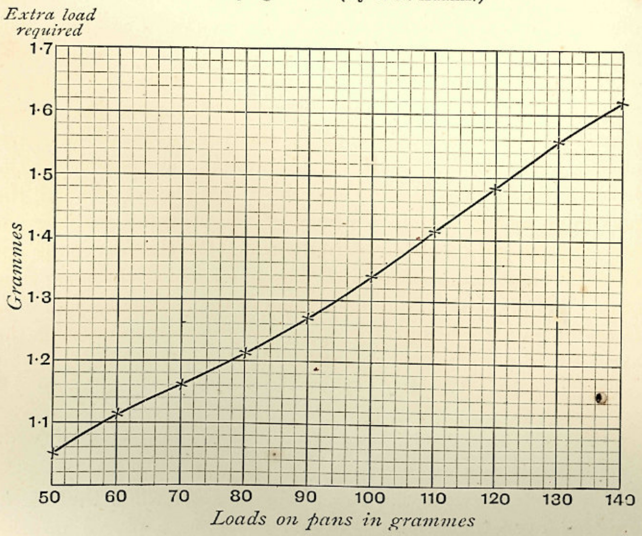


FIG. 144.—Curve showing similar observations made with a pulley of less accurate construction. (By R. B. Coare.)

smaller piece of wood, and the different values of the "pull" required to move the smaller piece of wood were observed and then plotted on the above diagram (Fig. 142). In order to measure these values, a hook was attached to the smaller piece of wood, and it was then dragged by the aid of a small spring balance, the value of the "pull" being read on the balance by one observer while the wood was being kept in slow motion by the other observer.

The friction of pulleys was also tested[1] under different loads, the loads being formed of equal masses attached to the two ends of a string passing over the pulleys. The value of the friction is given by the additional mass required to cause the pulley to move slowly. The results are recorded in the diagrams of Figs. 143 and 144.

AN EXAMPLE OF RECORDING VARIATIONS OF TEMPERATURE AND PRESSURE.

An illustration of a mode of recording variations of atmospheric pressure and temperature is given in Fig. 145. The observations were made during the month of November, 1893, at Tonbridge, and recorded for the School Natural History Society. It will be noticed that the barometer line is not necessarily an accurate record of the variations of pressure, although the crosses truthfully represent the height of the barometer at nine o'clock on each day. On the other hand, the maximum and minimum readings of the thermometer are not correctly recorded with regard to time, for the air could not have different temperatures at the same place at the same time of day as indicated. Nor, again, should the crosses be *joined by lines*, unless we regard the lines as useful in rendering the extremes of temperature on a given day more graphic.

[1] By J. D. Adams and R. B. Coare.

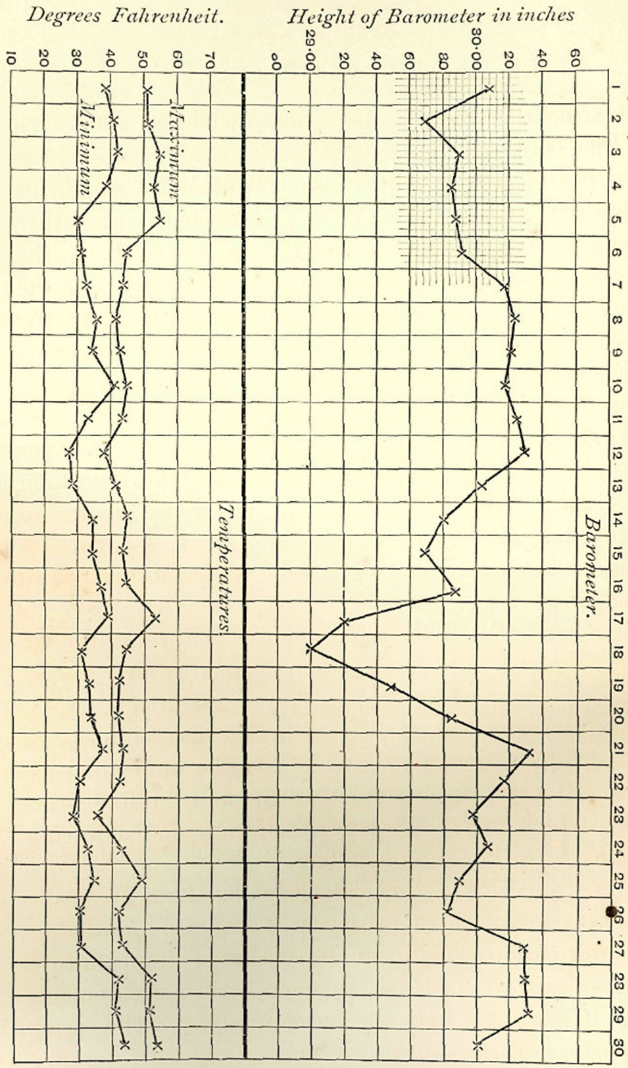


FIG 145.

A GENERAL SUMMARY.

During the whole of the course which is laid down in this book, one important lesson among others has been persistently instilled. At this stage we should be impressed by the fact that our knowledge of nature comes to us primarily through the channel of the senses. The observer must trust to his senses for the ground-work of knowledge. Observation through the senses, chiefly through that of sight, forms the basis of science. If the senses yield incorrect impressions, then our knowledge of nature is incorrect.

That all our knowledge is relative has been another of the great principles imparted. We know nothing of length, mass, and time, except with regard to another length, mass, or time. Each of these quantities must be measured by comparison with another of the same kind.

It is true that there are other aspects of nature, but they, too, must in some degree be gained from a knowledge of these three fundamental quantities. We learn from physical science itself that its confines are limited, and necessarily limited. At the same time we learn that there are certain methods and guiding principles which may be safely followed in acquiring knowledge within those confines.

The acquisition of this knowledge has been by means of measurement, the main source of accurate knowledge. All information which can be gathered from nature contains within itself the elements of measurement, even if only of indirect measurement. The mere comparison of one object, or one event, with another implies some kind of measurement, whether of mass, length, or time. This is true even for the barest descriptions of matter, size, or date.

Chief among the lessons learnt from our observations is, that nature is not capricious and unstable, but uniform and constant in action; that the apparent confusion around us disappears in proportion as we investigate methodically and carefully; that similar causes produce similar effects; that matter, space, and time can be brought under strict rules of investigation, and that we know little of these subjects beyond what can be learnt from such investigation.

It will not be necessary to point out that the operations which have been carried on in the laboratory are of the same nature as those by which we extend our knowledge of the world at large. We have studied typical cases, which have been selected with a view to the illustration of methods. Observations which are worthless in themselves take a high value when they form an illustration of a method or an exercise in accuracy.

We have noticed that both direct and indirect modes of observation are possible, although some direct observation is always necessary. Calculations based upon a few direct observations have been shown to lead indirectly to a more extensive knowledge of the object or change observed. But calculation is nothing more than the application to special cases of what has been observed to be a general truth. In calculating the area of a circle from the length of its radius, we carry on an operation which has been observed to be applicable to all circles.

But however important it may be to master the rudiments of science by learning how to measure simple quantities, it is soon perceived that various considerations influence even simple dimensions. At the earlier stages these must be neglected; we have to learn to select one property or change and disentangle it from the rest. We must, for instance, reserve for a later stage the alterations, and the laws expressing the alterations, in the dimensions of bodies caused by change of temperature, and we must be content for the present with an incomplete knowledge of what is meant by equal quantities of matter.

The simplest observations, however, soon lead to those involving more than one measurement. We pass from single quantities to those made up of more than one quantity; and these complex quantities are to be regarded as containing two or more simple quantities. And we perceive that we may regard objects as fixed or as undergoing change. Hence, a new and important measurement is added to our list, namely, the measurement of change itself.

Without entering into the multitude of quantities which form collectively the total of physical science, it has been shown that area, position, and volume involve at least two linear measurements; that density involves a measurement of mass and volume; that speed requires both measurement of length and time, and so on.

Together with the methods of observation have been presented the modes of recording or communicating the results of observation; and this part of our instruction is at least as important as the other. We learn to use words and numbers as signs of objects or ideas, of which they must be the faithful representatives. Each word which is expressed should always stand for the same object, or should call up, in the minds of those to whom it is communicated, the same idea or thought.

It is in order to impress this important rule that scientific training takes the form of practical measurements which may be useless in themselves, or of a repetition of observations which may have been already performed a countless number of times. Although knowledge may appear to be acquired more slowly in this way than from verbal descriptions, it is acquired more soundly, and it is more completely mastered. We are thereby able to grasp the real meaning of words which are used by ourselves and others. We learn, in fact, to subordinate the signs to the realities, and escape that confusion between signs and realities which is so frequently encountered.

Moreover, it is by a practical acquaintance with the methods and instruments of observation, that we are able to extend our knowledge in directions which have been so far unexplored. In other words, it is a training which should enable us to carry on original research.

In the use of diagrams we have an extension of our means of record, and by their aid the chance of confusion of meaning is diminished; while they convey facts, or rather the results of observations, to the mind in the most direct manner by means of relative lengths and positions. It must not be supposed that diagrams enable us to dispense with words and numbers; they do nothing more than convey to us words and numbers in a shortened form. A diagram without an explanation in words of what it is meant to express does not constitute a record. Hence it is important that every diagram should bear on its face a description of its meaning, and the values of its marks and distances.

After records, whether in the form of tables of numbers or diagrams, must come the inferences

which may be drawn from them. We must first of all learn to observe correctly. Afterwards we draw conclusions or build theories on our observations. We shall then probably escape that common error of supposing that a fact is "explained" when it is expressed by two equivalent terms. We cannot explain the fall of a body to the earth by saying it is due to gravitation; nor do we explain the floating of wood in water by speaking of the buoyancy of water. No additional information is conveyed by a mere exchange of words.

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